Image derivatives, edges, corners, . . .

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Talk Outline

- why corners?
- combined edge and corner detector
- Demo: step by step in Matlab

Dense tracking

Video: Dense tracking. Note that the coverage is not complete

What are good features (windows) to track?

How to select good templates $T(\mathbf{x})$ for image registration, object tracking.

$$\Delta\mathbf{p} = \mathtt{H}^{-1} \sum_{\mathbf{x}} \left[\nabla I \, \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^{\top} [T(\mathbf{x}) - I(\mathbf{W}(\mathbf{x};\mathbf{p}))]$$

where $\mathtt{H}$ is the Hessian matrix

$$\mathtt{H} = \sum_{\mathbf{x}} \left[\nabla I \, \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^{\top} \left[\nabla I \, \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]$$

The stability of the iteration is mainly influenced by the inverse of Hessian. We can study its eigenvalues. Consequently, the criterion of a good feature window is $\min(\lambda_1, \lambda_2) > \lambda_{min}$ (texturedness).

What are good features (windows) to track?

Consider translation $\mathbf{W}(\mathbf{x};\mathbf{p}) = \begin{bmatrix} x + p_1 \\ y + p_2 \end{bmatrix}$. The Jacobian is then $\frac{\partial \mathbf{W}}{\partial \mathbf{p}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\begin{aligned}
\mathtt{H} &= \sum_{\mathbf{x}} \left[\nabla I \ \frac{\partial \mathbf{W}}{\partial \mathbf{p}}\right]^\top \left[\nabla I \ \frac{\partial \mathbf{W}}{\partial \mathbf{p}}\right] \\
&= \sum_{\mathbf{x}} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\partial I}{\partial x} \\ \frac{\partial I}{\partial y} \end{bmatrix} \left[\frac{\partial I}{\partial x}, \frac{\partial I}{\partial x}\right] \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
&= \sum_{\mathbf{x}} \begin{bmatrix} \left(\frac{\partial I}{\partial x}\right)^2 & \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} \\ \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} & \left(\frac{\partial I}{\partial y}\right)^2 \end{bmatrix}
\end{aligned}$$

The image windows with varying derivatives in both directions. Homeogeneous areas are clearly not suitable. Texture oriented mostly in one direction only would cause instability for this translation.

What are the good points for translations?

The Hessian matrix

$$\mathtt{H} = \sum_{\mathbf{x}} \begin{bmatrix} \left(\frac{\partial I}{\partial x}\right)^2 & \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} \\ \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} & \left(\frac{\partial I}{\partial y}\right)^2 \end{bmatrix}$$

Should have large eigenvalues. Harris and Stephens were motivated differently, yet they ended with their famous[1] corner detector [1].

[1] 3635 times cited according to Google scholar on 2009-10-26.

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Few questions arise:

- why corners?
- what was the motivation for corners?
- what are corners, actually?

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Image matching

- uniqueness
- repeatability

Derivation of the corner detector on the blackboard. Details can be found in [1]. Its implementation with demonstration of use in chapter 5 of [2].

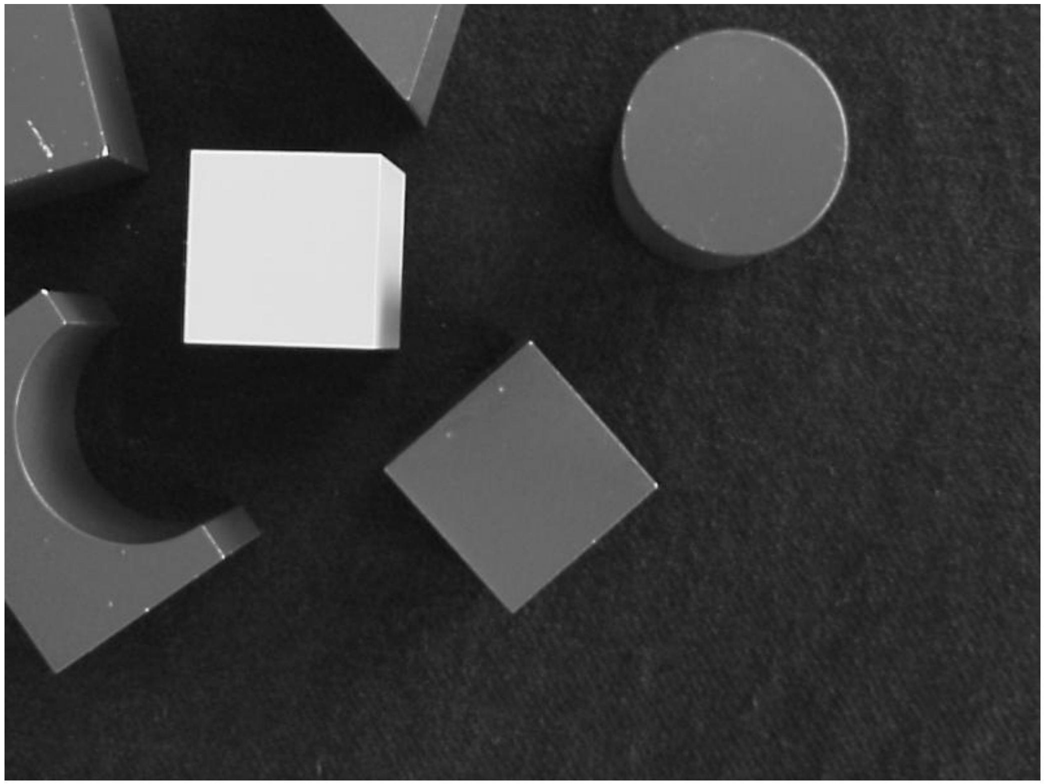
Image derivatives

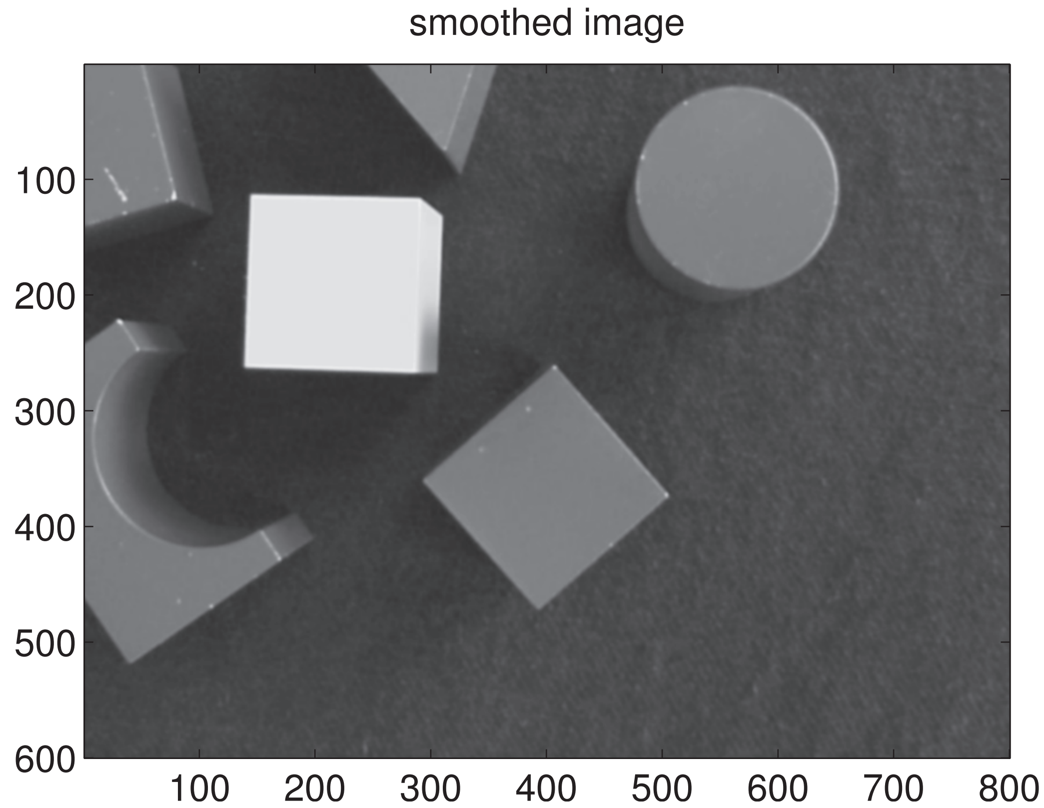
In discrete space we aproximate them by local differences:

$$
\begin{aligned}
\frac{\partial I}{\partial x}(x,y) &= I(x+1,y) - I(x-1,y) \\
\frac{\partial I}{\partial y}(x,y) &= I(x,y+1) - I(x,y-1)
\end{aligned}
$$

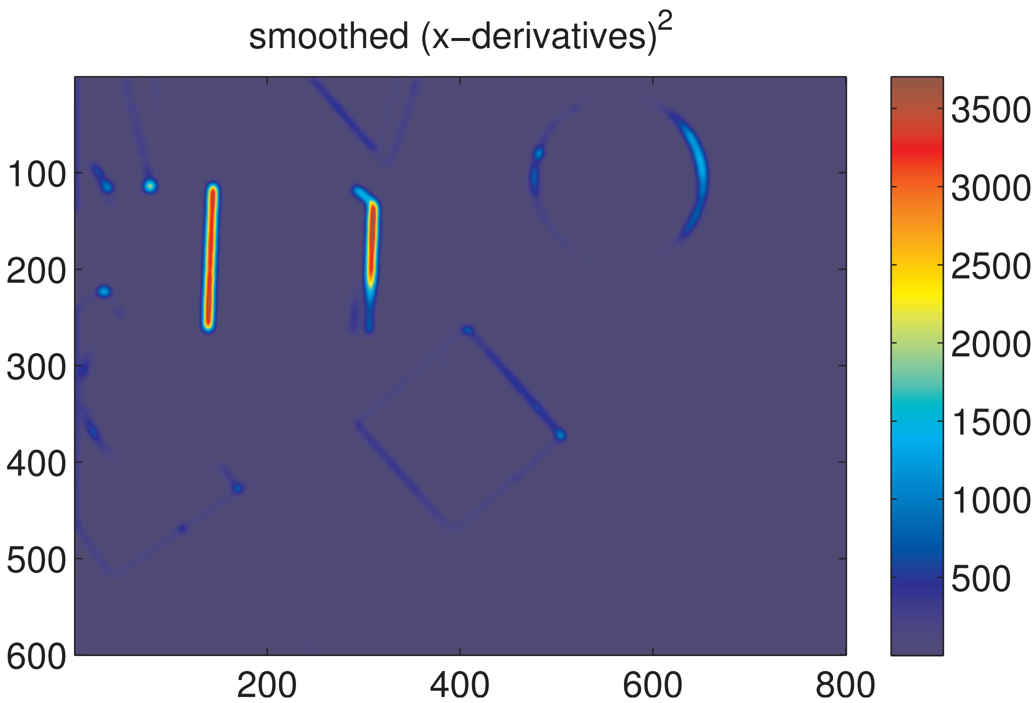
See the on-line demonstration in Matlab

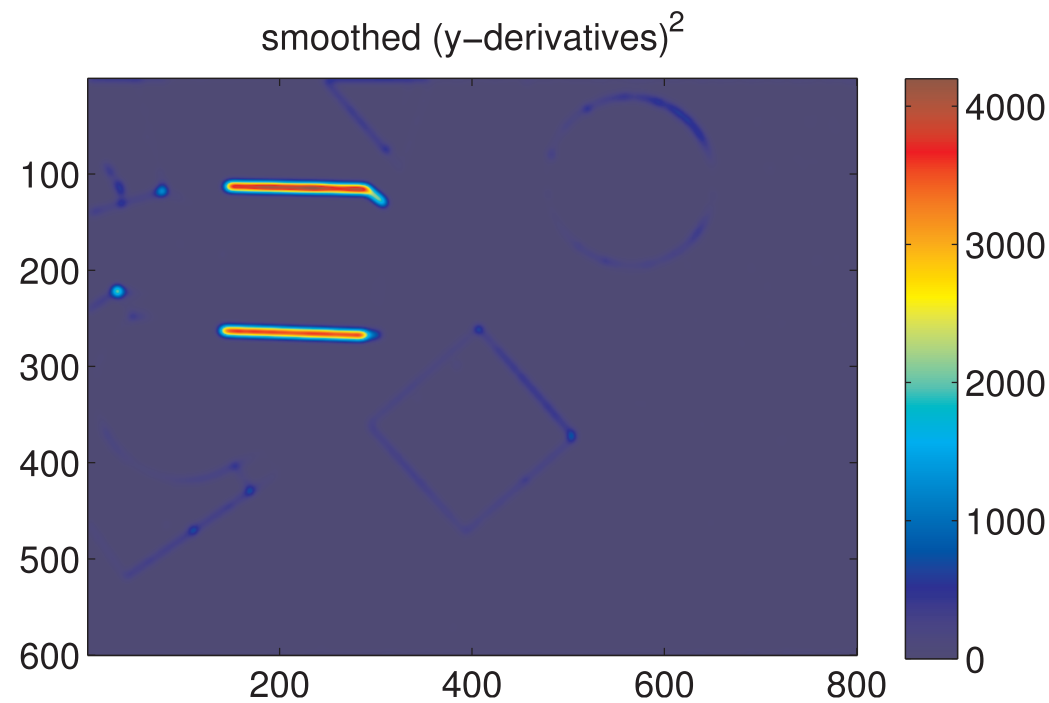
Corner detection in pictures, images

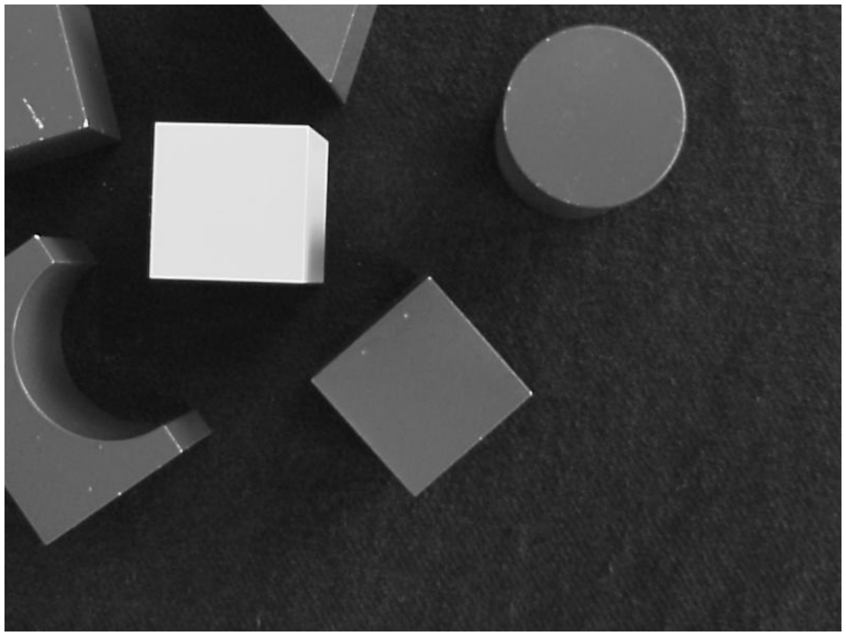




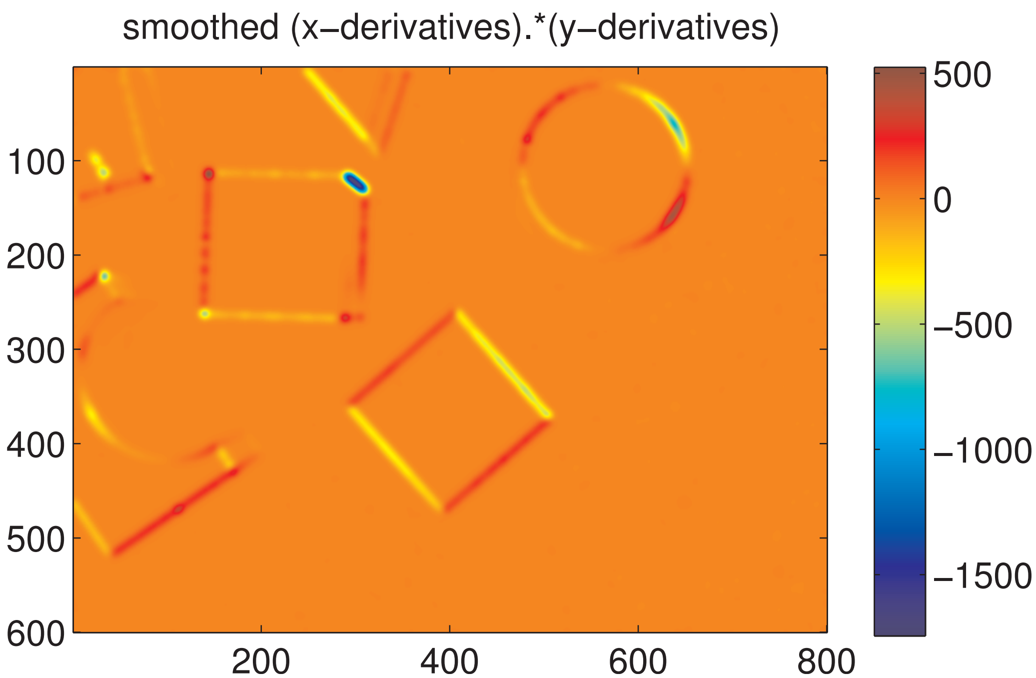
Corner detection in pictures, (smoothed) derivatives

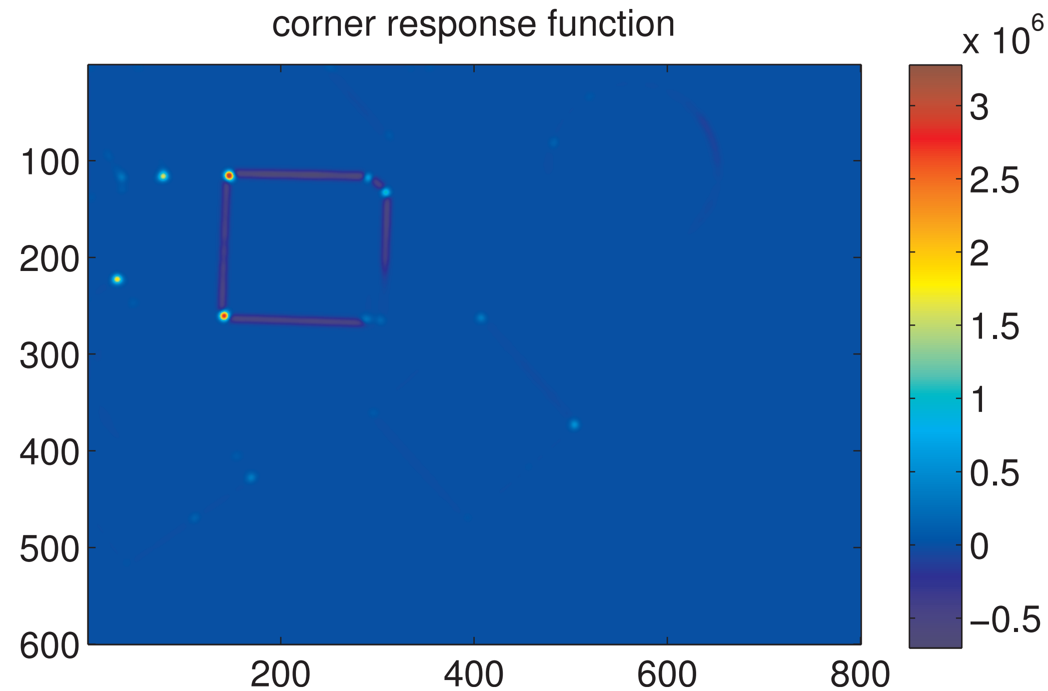


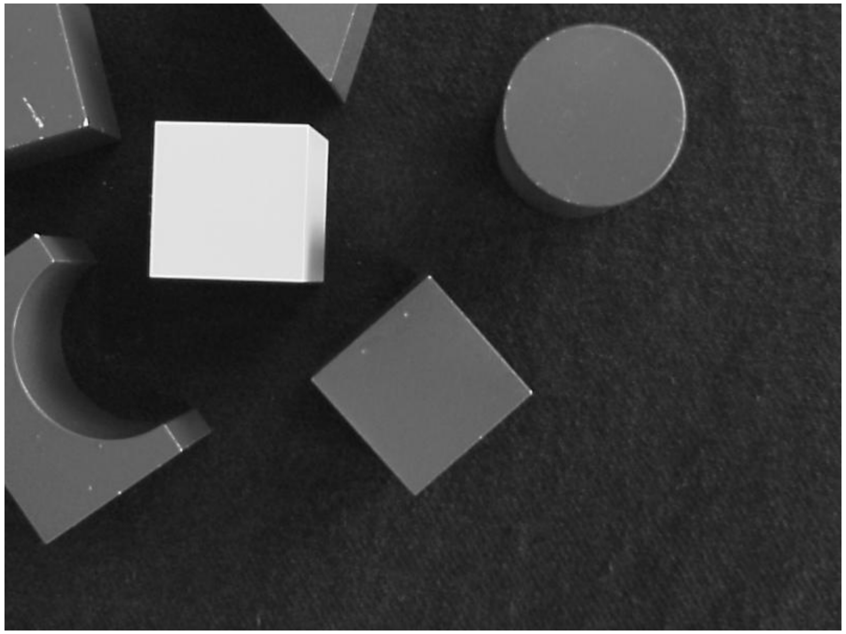




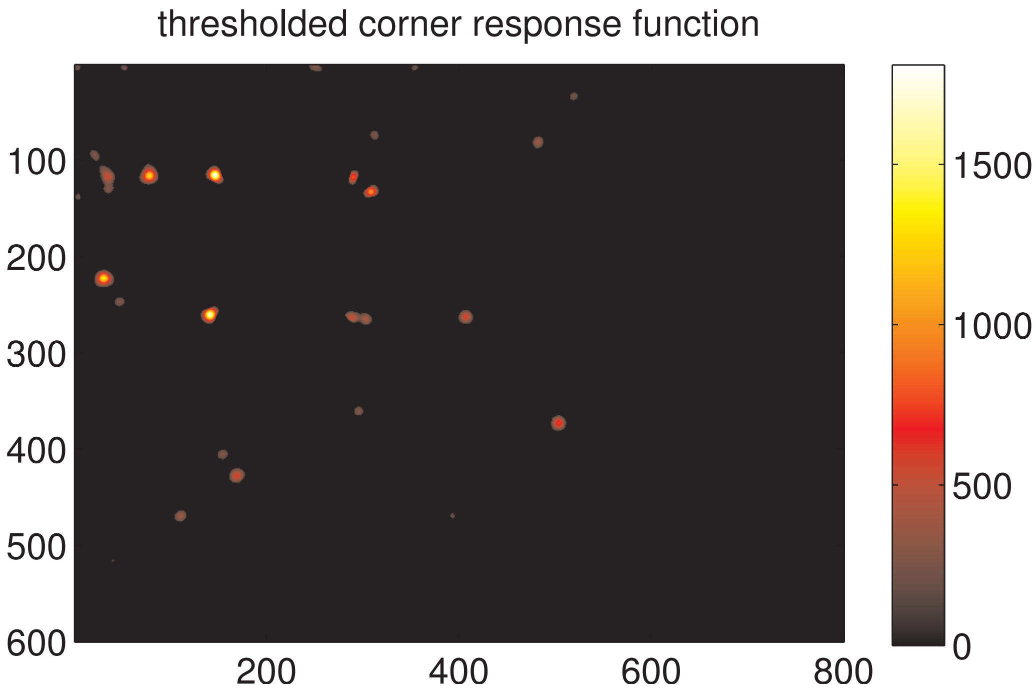
Corner detection in pictures, mixed derivatives and reponse function

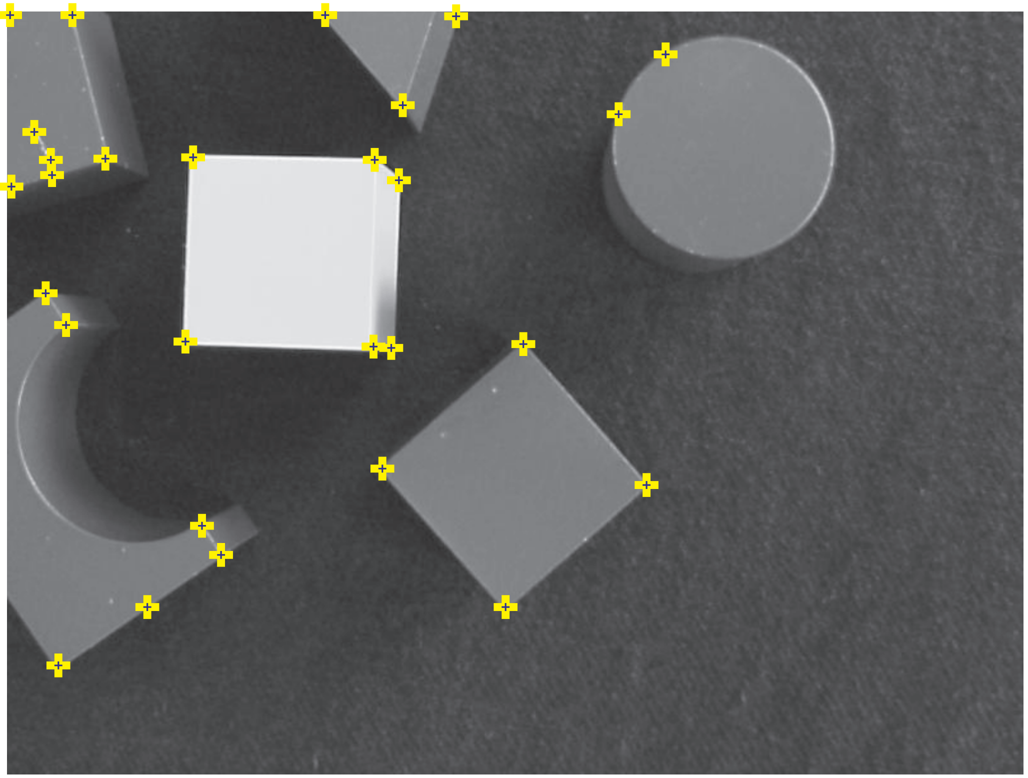






Corner detection in pictures, thresholding and corners



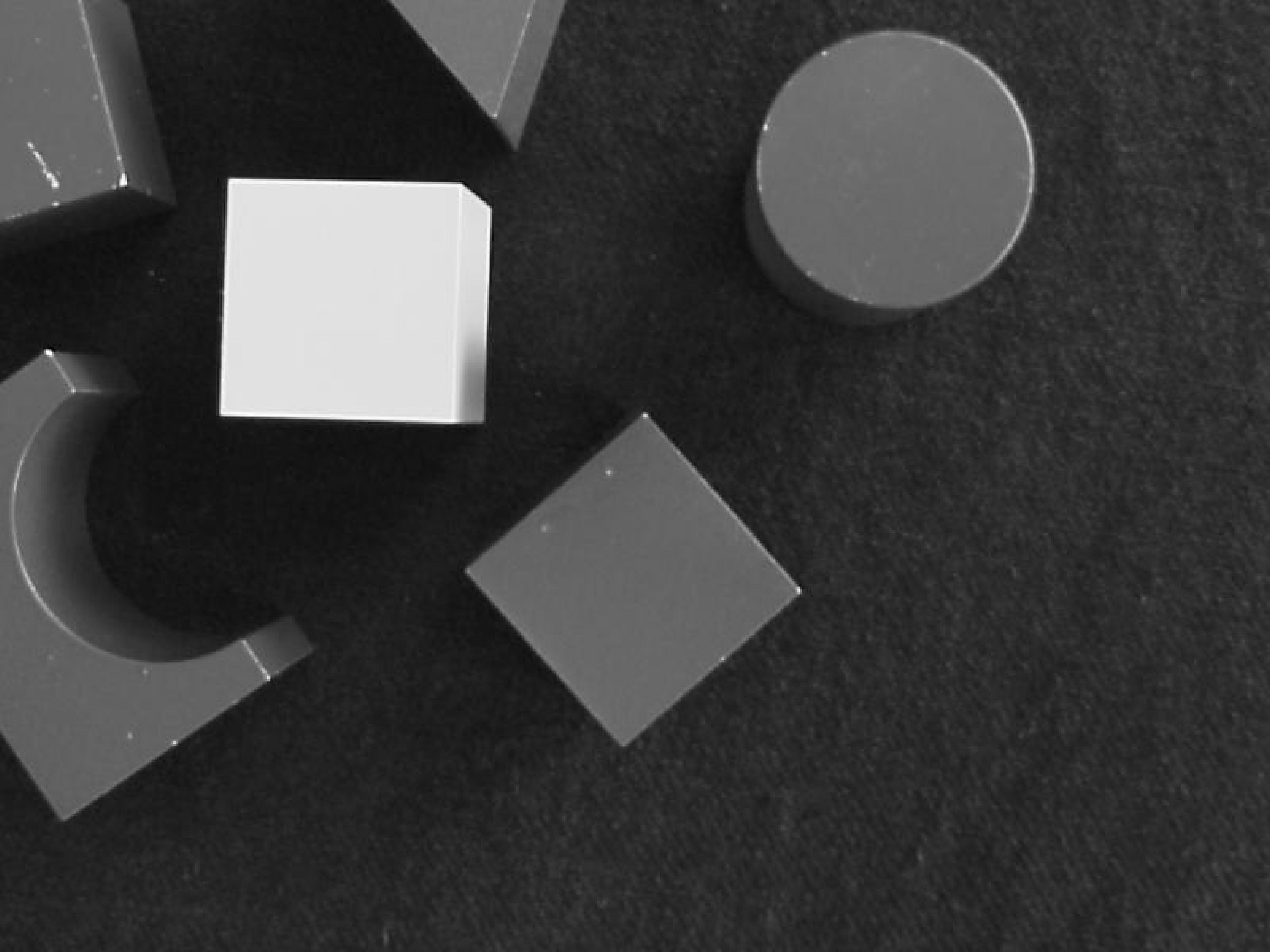


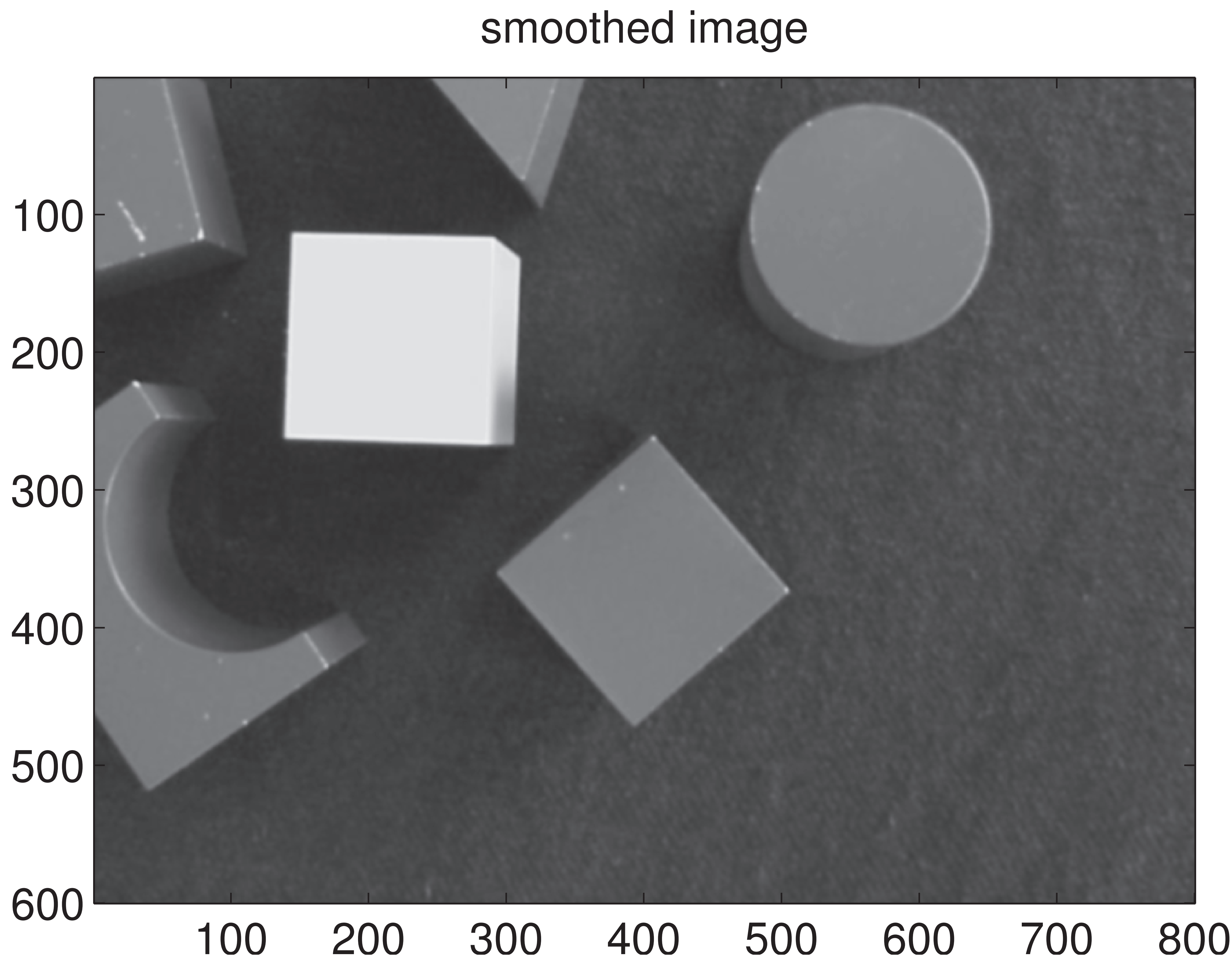
References

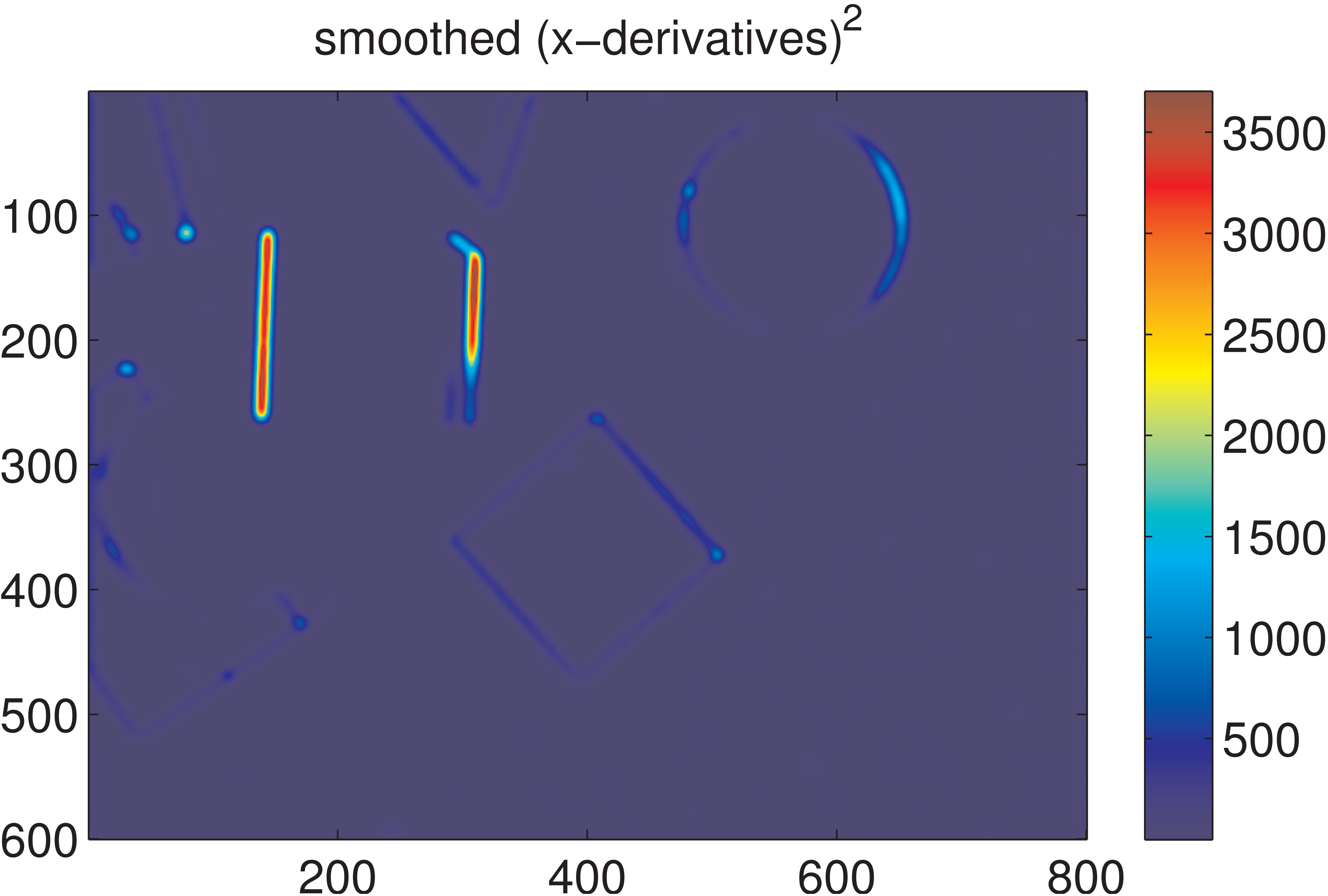
[1] C. Harris and M. Stephen. A combined corner and edge detection. In M. M. Matthews, editor, Proceedings of the 4th ALVEY vision conference, pages 147–151, University of Manchaster, England, September 1988. on-line copies available on the web.

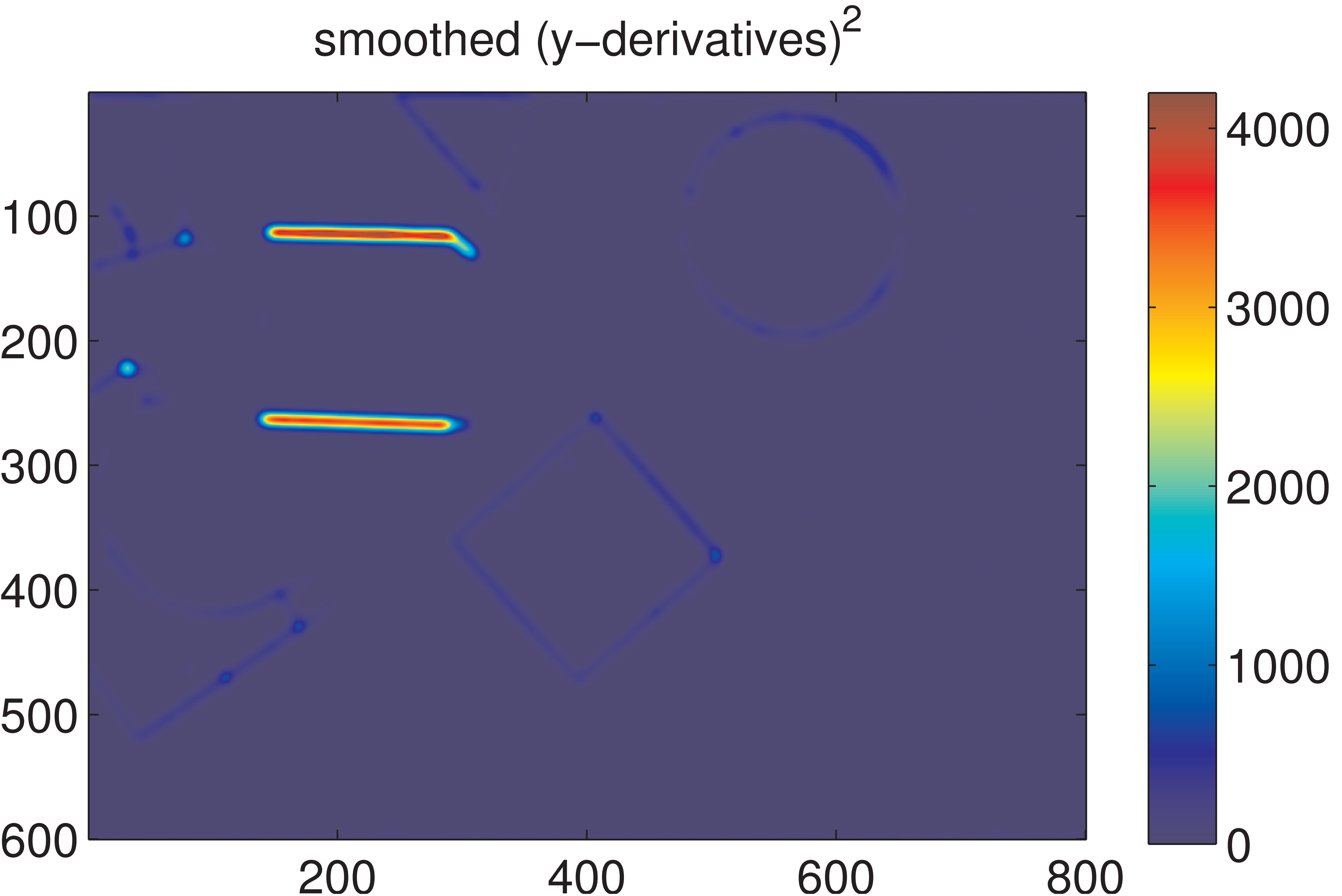
[2] Tomáš Svoboda, Jan Kybic, and Václav Hlaváč. Image Processing, Analysis and Machine Vision. A MATLAB Companion. Thomson, 2007. Accompanying www site http://visionbook.felk.cvut.cz.

End

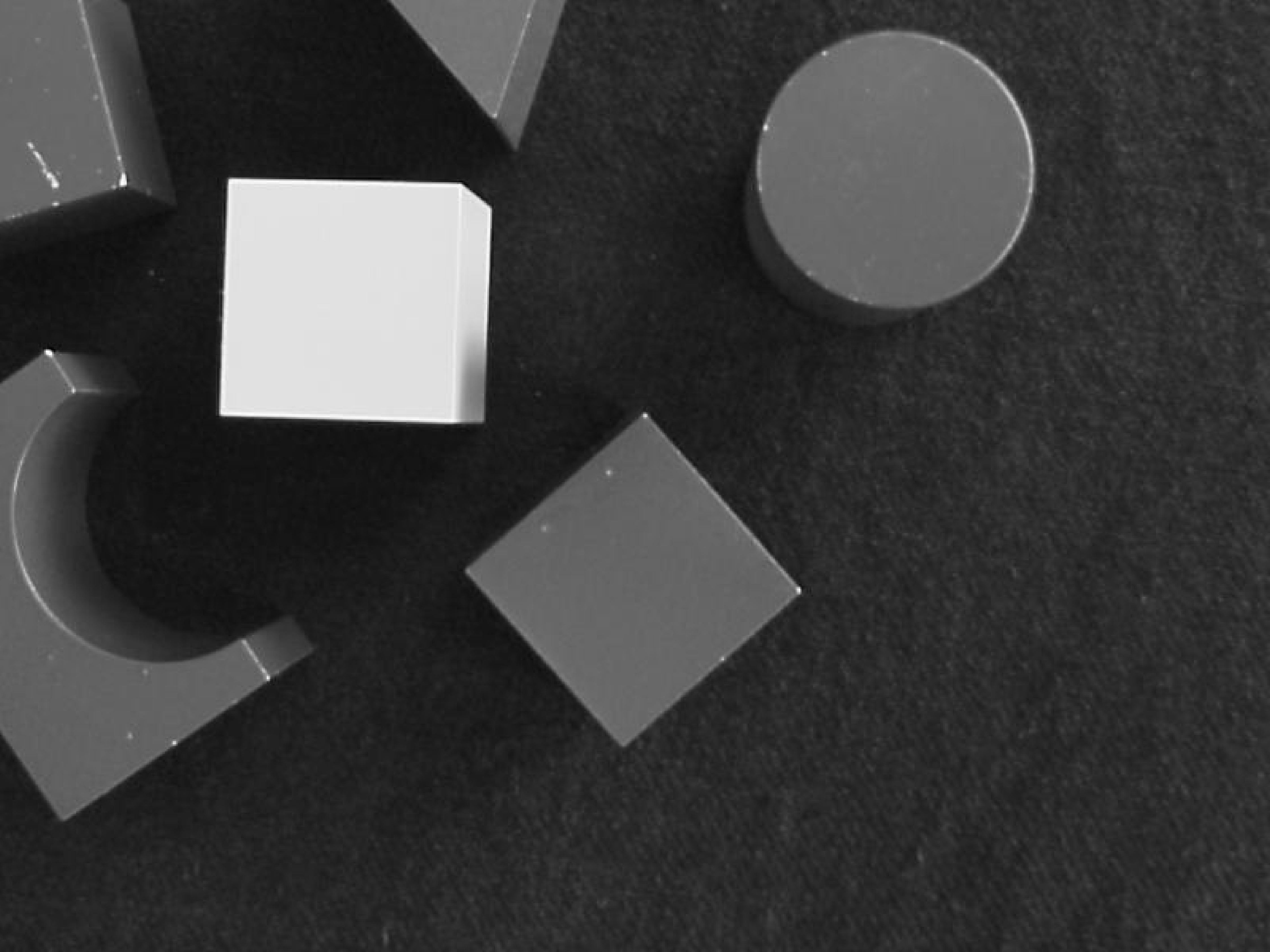


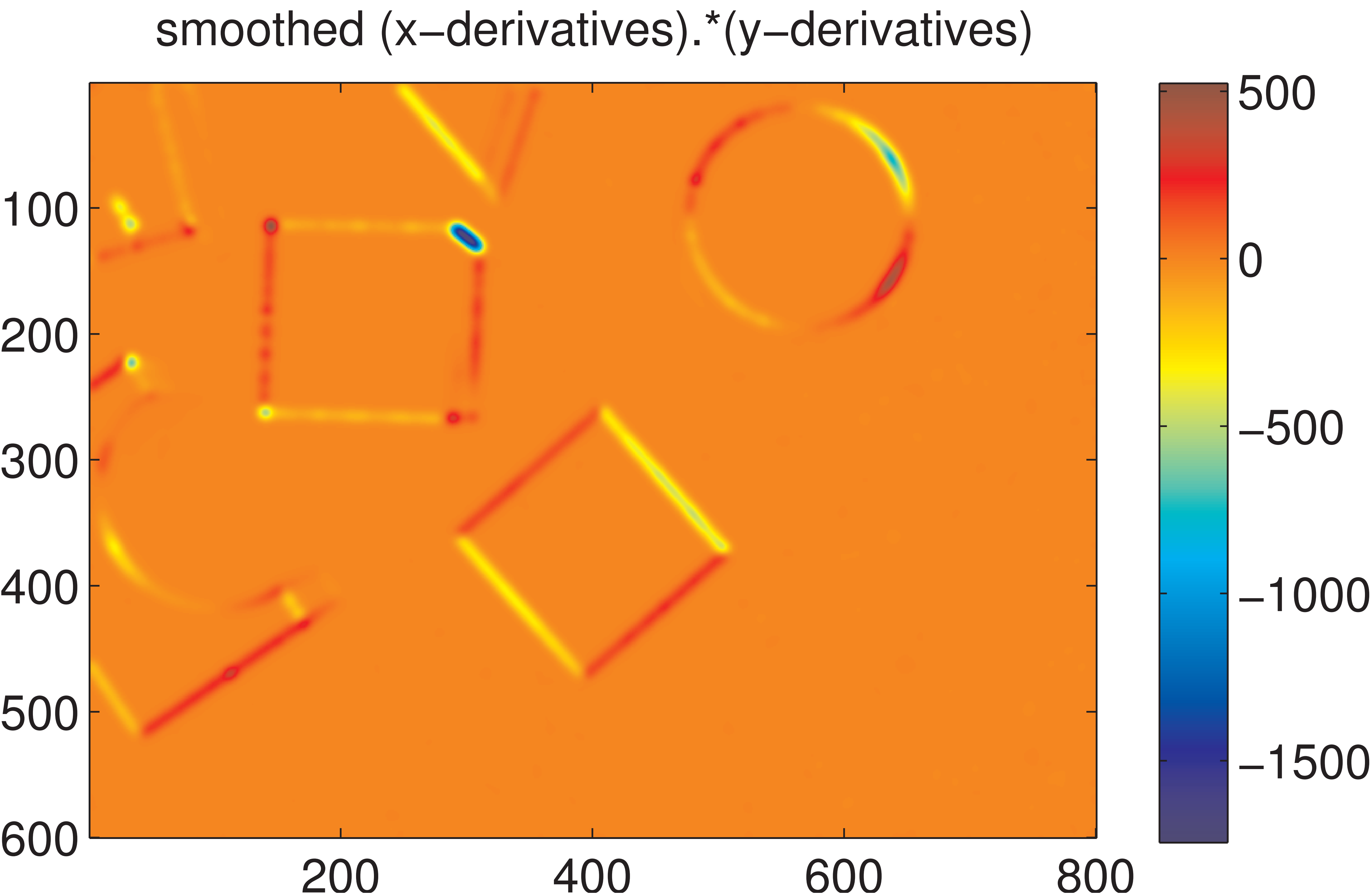

smoothed image
100
200
300
400
500
600
100
200
300
400
500
600
700
800

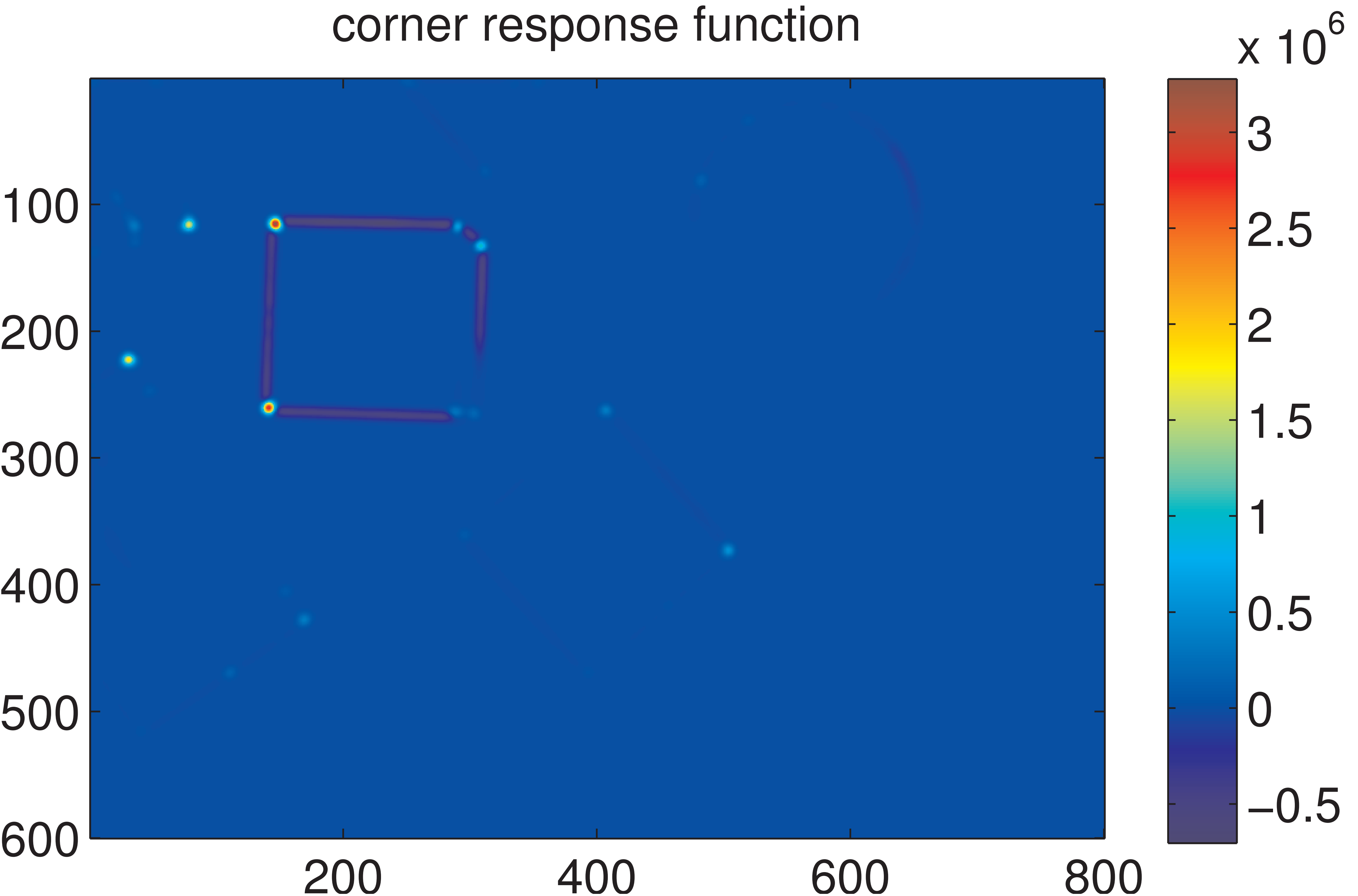

smoothed (x–derivatives)^2
100
200
300
400
500
600
200
400
600
800
3500
3000
2500
2000
1500
1000
500

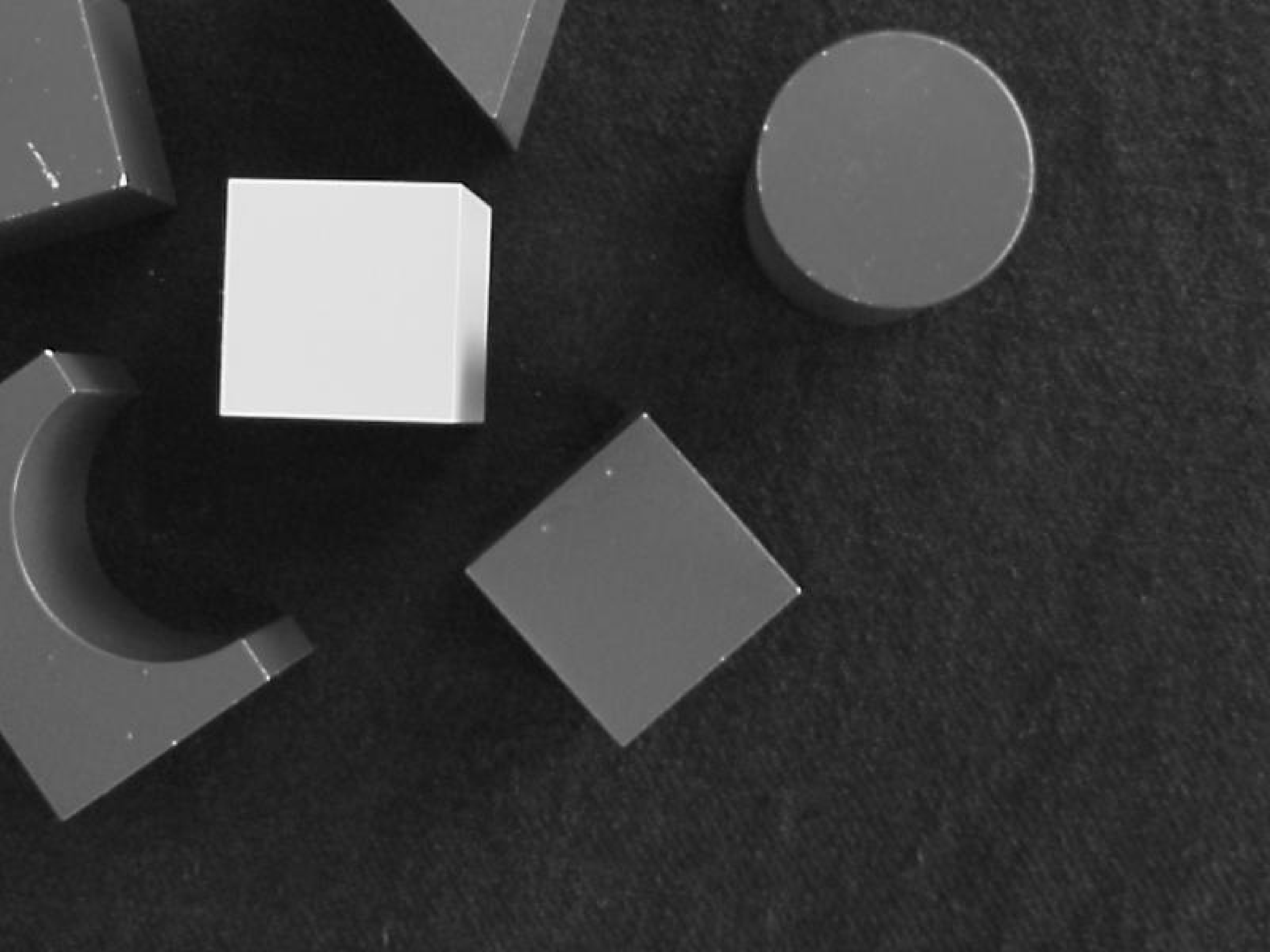


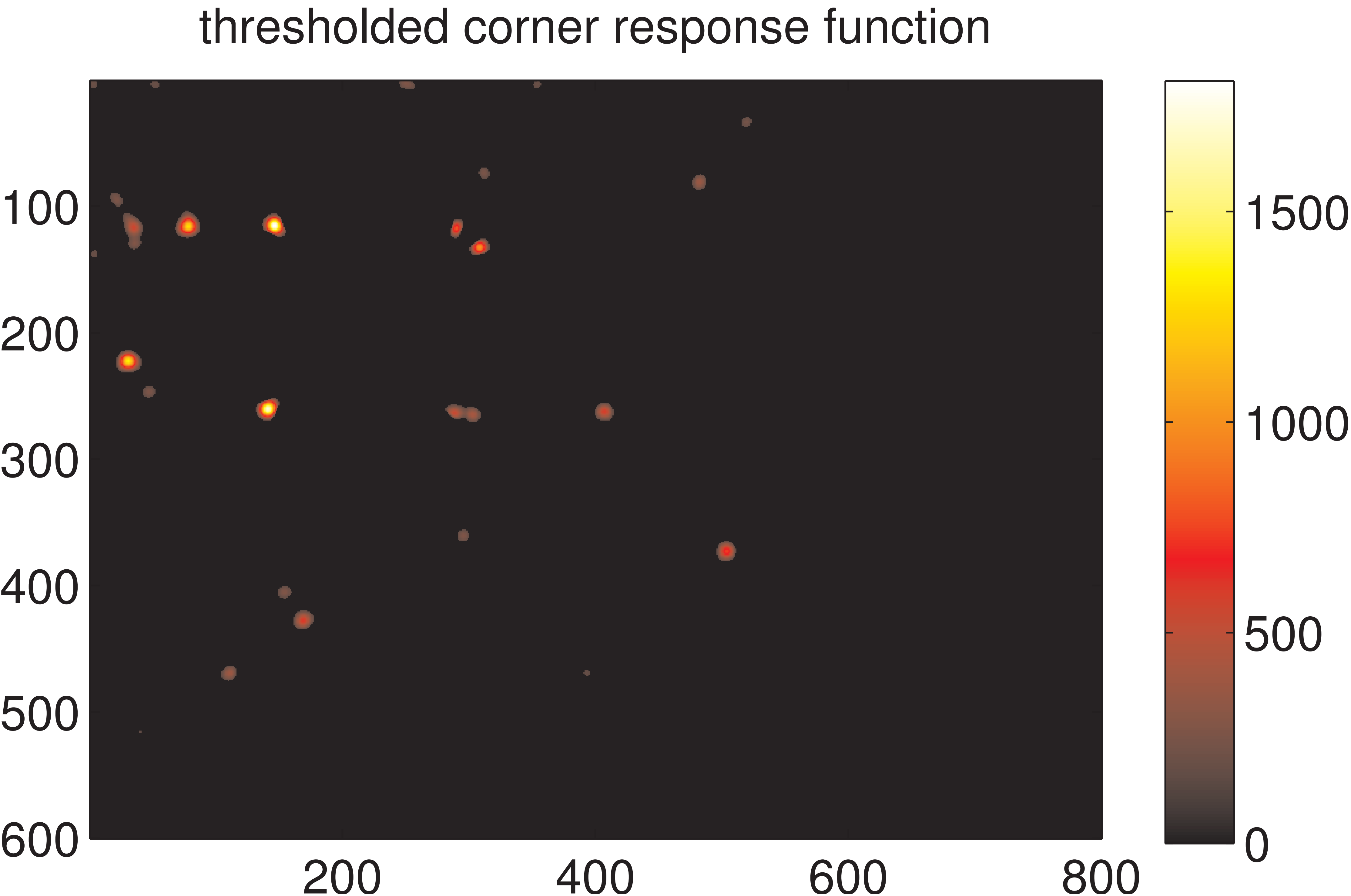
smoothed (y−derivatives)^2
100
200
300
400
500
600
200
400
600
800
0
1000
2000
3000
4000




smoothed (x−derivatives).*(y−derivatives)
100
200
300
400
500
600
200
400
600
800
500
0
−500
−1000
−1500


corner response function
x 10^6
3
2.5
2
1.5
1
0.5
0
−0.5
100
200
300
400
500
600
200
400
600
800




thresholded corner response function
100
200
300
400
500
600
200
400
600
800
1500
1000
500
0