

$J_\theta(p)$ is Radon transform of image $f(x, y)$

$$J_\theta(p) = \int_{-\infty}^{\infty} f(p \cos \theta - q \sin \theta, p \sin \theta + q \cos \theta) dq,$$

where

$$\begin{aligned} p &= x \cos \theta + y \sin \theta \\ q &= -x \sin \theta + y \cos \theta \\ x &= p \cos \theta - q \sin \theta \\ y &= p \sin \theta + q \cos \theta. \end{aligned} \tag{1}$$

Easy way to reconstruct the image $f(x, y)$ is to use backprojection (2). Reconstructed image $\hat{f}(x, y)$ is computed as

$$\hat{f}(x, y) \approx \int_0^\pi J_\theta(p) d\theta. \tag{2}$$

Note that this is only approximate reconstruction of the image f .

The second option is to reconstruct the image $f(x, y)$ using a filtered backprojection. Let $S_\theta(\omega)$ be the 1D Fourier transform of the projection $J_\theta(p)$

$$S_\theta(\omega) = \int_{-\infty}^{\infty} J_\theta(p) e^{-j2\pi\omega p} dp. \tag{3}$$

The reconstructed image \hat{f} is computed using backprojection (2) of the filtered projection $J_\theta^*(p)$

$$f(x, y) = \int_0^\pi J_\theta^*(p) d\theta, \tag{4}$$

where $J_\theta^*(p)$ is the projection $J_\theta(p)$ after filtering

$$f(x, y) = \int_0^\pi \int_{-\infty}^{\infty} |\omega| S_\theta(\omega) e^{j2\pi\omega(x \cos \theta + y \sin \theta)} d\omega d\theta \tag{5}$$

$$\begin{aligned} J_\theta^*(p) &= \int_{-\infty}^{\infty} |\omega| S_\theta(\omega) e^{j2\pi\omega(x \cos \theta + y \sin \theta)} d\omega \\ &= \int_{-\infty}^{\infty} |\omega| S_\theta(\omega) e^{j2\pi\omega p} d\omega \\ &= \mathcal{F}^{-1}\{|\omega| S_\theta(\omega)\} \\ &= \mathcal{F}^{-1}\{|\omega|\} \otimes \mathcal{F}^{-1}\{S_\theta(\omega)\} \\ &= \mathcal{F}^{-1}\{|\omega|\} \otimes J_\theta(p) \\ &= \mathcal{F}^{-1}\{|\omega| \mathcal{F}\{J_\theta(p)\}\}, \end{aligned} \tag{6}$$

where \mathcal{F} is the symbol for Fourier transformation and \otimes is the convolution operator.