Two Alternative Problem Formulations

Simplifications:

1. Matching is done independently per epipolar lines.
2. Ordering holds.
3. Data in occluded regions is irrelevant.

Overview

1. Optimal Bayesian decision with unit decision cost $\longrightarrow$ MAP estimate

   Algorithm: Minimum-cost path in a directed circuit-free graph

   (via dynamic programming with fixed ends)

2. Stable matching

   Algorithm: Strict sub-kernel of a directed graph

- These are just two formulations of many.

  No good review exists, especially a good algorithm review.

Bayesian Decision Task for Matching

Idea: $L(d, M)$ – decision cost (loss) d – our decision M – true matching (unknown)

decision cost is application-oriented

Choice: $L(d, M) : \begin{cases} \text{if } d = M & \text{then } L(d, M) = 0 \\ \text{if } d \neq M & \text{then } L(d, M) = 1 \end{cases}$ i.e. $L(d, M) = [d \neq M]$

Bayesian Loss

$L(d|X)$

$$L(d) = \sum_{M \in \mathcal{M}} p(M \mid X)\, L(d, M)$$

$\mathcal{M}$ – the set of all matchings X – data, $X = \{I_L, I_R\}$

Problem

$$M^* = \arg\min_d \sum_{M \in \mathcal{M}} p(M \mid X)\,(1 - [d = M]) = \arg\min_d \left(1 - \sum_{M \in \mathcal{M}} p(M \mid X)[d = M]\right) =$$

$$= \arg\max_d \sum_{M \in \mathcal{M}} p(M \mid X)\,[d = M] = \arg\max_M p(M \mid X)$$

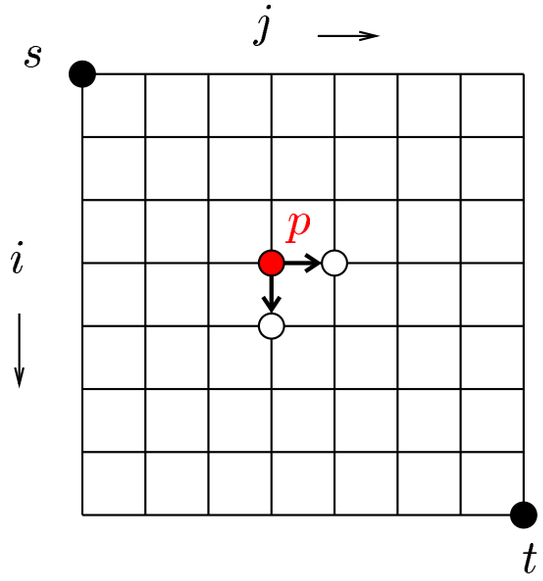
$$= \arg\min_M (-\log p(M \mid X)) = \arg\min_M V(M \mid X)$$

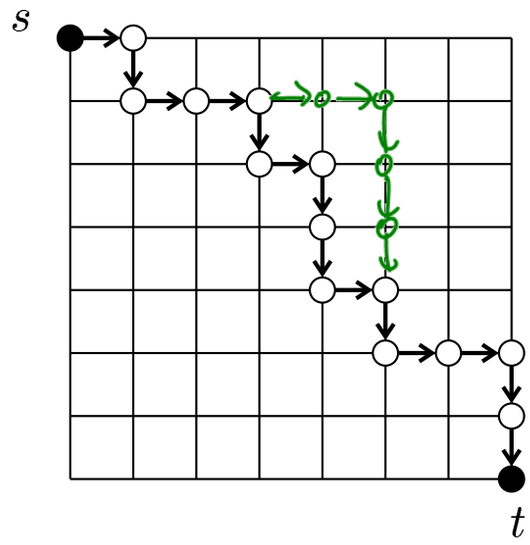
Problem solution: Maximum Aposteriori Probability (MAP) estimate via energy minimization methods

Constructing the Matching Task

$$M^* = \arg\min_{M \in \mathcal{M}} V(M \mid X) = \arg\min_{M \in \mathcal{M}} V(X \mid M) + V(M)$$

- We need a representation to encode $V(M)$
  - Every $p = (i, j)$ of the matching table T (except for the last column) receives two succesors $S(p)$ including $(i + 1, j)$ and $(i, j + 1)$





the circles are not yet matches!

  - This gives a directed graph $\mathcal{G}$ with no circuit optimal paths in acyclic graphs are an easier problem
  - The set $\mathcal{P}$ of s-t paths starting in s and ending in t represent the set $\mathcal{M}$ of matchings
  - All such paths have **equal** length $|I| + |J| - 1$ will

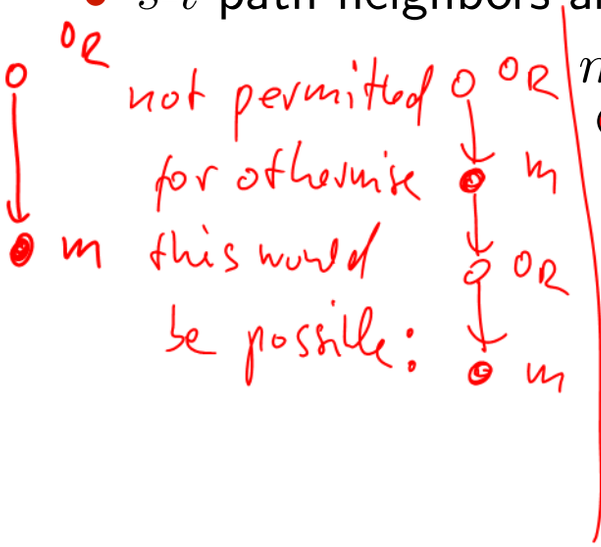
all prospective matchings have the same number of terms in $V(M)$

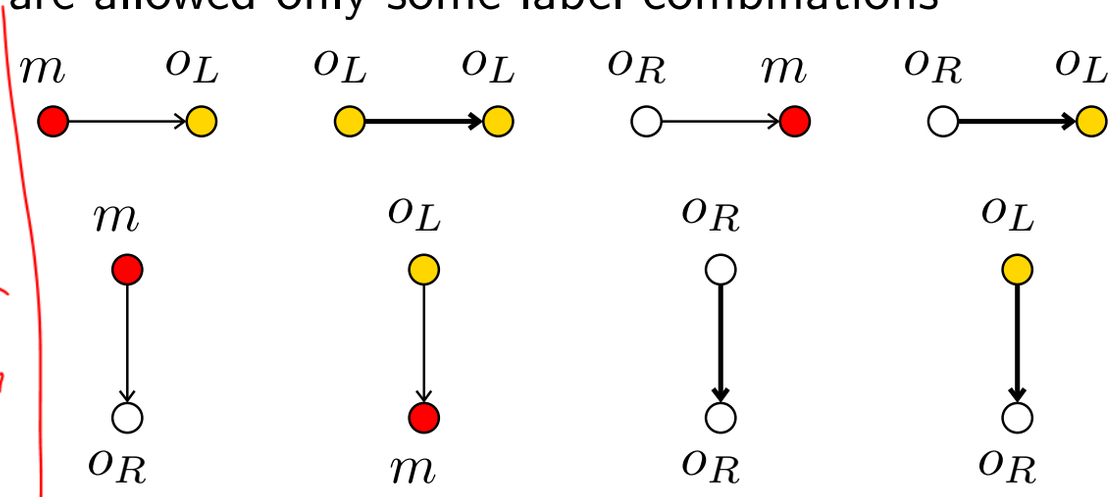
Endowing s-t Paths with Useful Properties

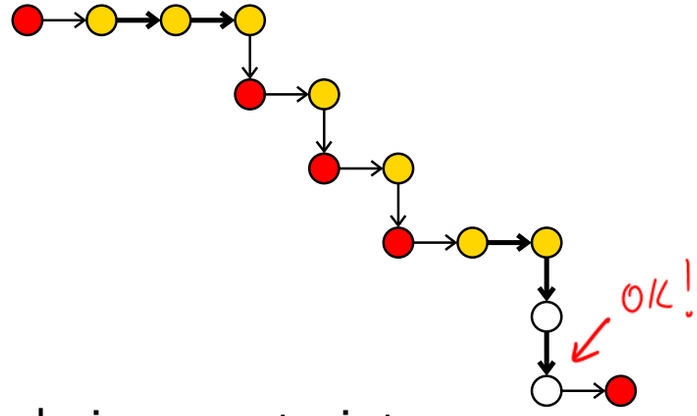
- introduce node labels $\Lambda = \{m, o_L, o_R\} = \{m\} \cup \Omega$

  matched, occluded in left img, occluded in right img

- s-t path neighbors are allowed only some label combinations







Observations

- no two neighbors have label m
- in each s-t path there is at most one transition:
  1. $m \longrightarrow o_L$ or $o_R \longrightarrow m$ per matching table row,
  2. $m \longrightarrow o_R$ or $o_L \longrightarrow m$ per matching table column
- pairs labeled m on every s-t path satisfy uniqueness and ordering constraints
- transitions $o_L \longrightarrow o_R$ or $o_R \longrightarrow o_L$ along an s-t path correspond to skipping a contiguous segment in either or in both images

  this models half occlusion and mutual occlusion

Energy Function Choice

$$V(M \mid X) = V(X \mid M) + V(M) = \alpha_0 \sum_{p \in M, q \in S(p)} V_{12}(p, q) + \sum_{p \in M} V_1(p)$$

- *choose* stationary $V_{12}\big(p, \lambda(p), q, \lambda(q)\big) = V_{12}\big(\lambda(p), \lambda(q)\big)$ proportional to disparity between two successive nodes of label $o \in \Omega$
- occluded segments $m \to o \to \cdots \to o \to m, \quad o \in \Omega$ then contribute to $\sum V_{12}\big(\lambda(q), \lambda(p)\big)$ a factor proportional to the length of the segment
- use Markov chains to find the optimal solution
- *choose* $V_1\big(p, \lambda(p)\big) = V_1\big(p \mid \lambda(p)\big) + V_0$

The matching problem becomes

$$M^* = \arg\min_{M \in \mathcal{P}} \underbrace{\sum_{p \in M,\ q \in S(p)} V_{12}\big(\lambda(q) \mid \lambda(p)\big)}_{\text{over } s\text{-}t \text{ path edges}} + \underbrace{\sum_{p \in M} V_1\big(p \mid \lambda(p)\big)}_{\text{over } s\text{-}t \text{ path nodes}}$$

- $\mathcal{P}$ is the set of all s-t paths
- $S(p)$ are immediate successors of p along an s-t path
- segments $m \to o_L \to m$, $m \to o_R \to m$ have no cost in V_{12}
- α_0 prior model weight
- $\alpha_1 = \frac{\Pr(o_L|o_R)}{\Pr(o_L|o_L)} \geq 0$: ratio of transition probability of mutual occlusion to half-occlusion ($\alpha_1 = 0$ forbids mutual occlusion)

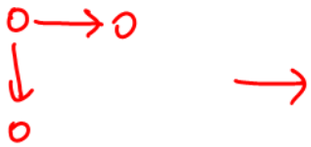
$P(o_R|o_R) = P(o_L|o_L)$

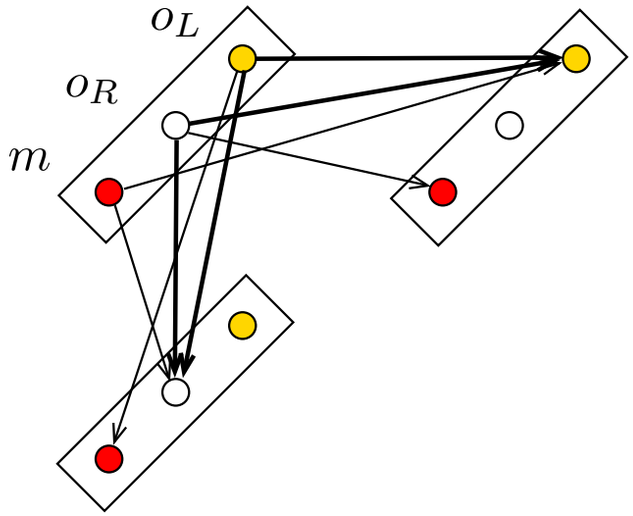
$\lambda(p)$	$\lambda(q)$	$V_{12}\big(\lambda(q), \lambda(p)\big)$
m	o_L	0
o_L	o_L	$\alpha_0 \ln(1+\alpha_1)$
o_R	m	0
o_R	o_L	$\alpha_0 \ln(1+\alpha_1^{-1})$
m	o_R	0
o_L	m	0
o_R	o_R	$\alpha_0 \ln(1+\alpha_1)$
o_L	o_R	$\alpha_0 \ln(1+\alpha_1^{-1})$

$\lambda(p)$	$V_1\big(p \mid \lambda(p)\big)$
m	image similarity
o	α_2

Solving the Matching Problem

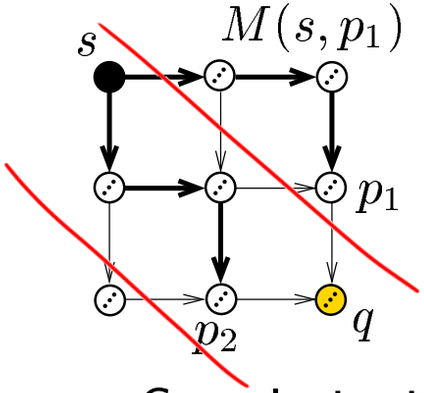
- Construct directed graph $\mathcal{G}^+$, triple of vertices per node of s-t path representing three hypotheses $\lambda(p)$ for $\lambda \in \Lambda$
- Orientation of $\mathcal{G}^+$ is inherited from the orientation of s-t paths





strong edges are of non-zero penalty

- We can now use **dynamic programming** algorithm on $\mathcal{G}^+$



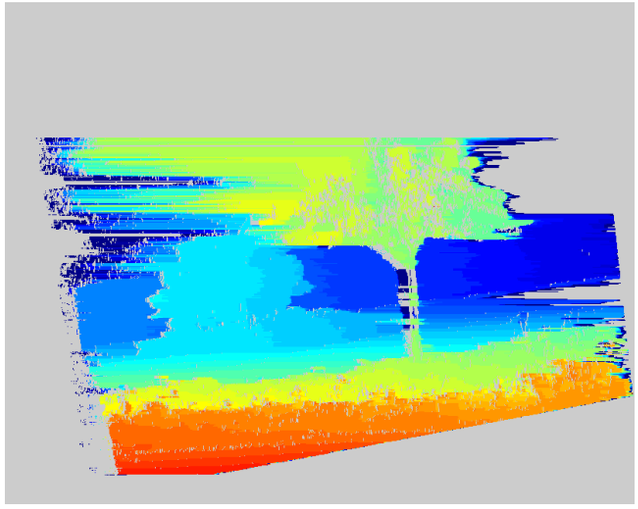
$$V\big(M(s,p_1)\big) + V_1(p_1) + V_{12}(p_1,q) \overset{?}{<} V\big(M(s,p_2)\big) + V_1(p_2) + V_{12}(p_2,q)$$

- Complexity is $O(|I| \cdot |J|)$, ie. stereo matching on $N \times N$ images needs $O(N^3)$ time
- Speedup by limiting the range in which the disparities $d = j - i$ are allowed to vary

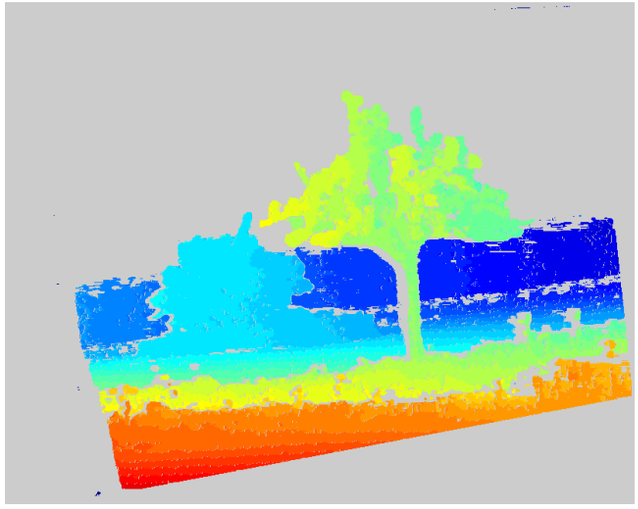
Matching Results

left image

right image



MAP with DP algorithm



Strict Sub-Kernel algorithm (next)

- MAP can be improved by controlling prior model weight α_0 by image edges
- The principal problem with MAP: instability of the solution

Energy Minimizers are Not Always Stable

- Consider a perfect bipartite matching problem
- Solutions shown in gold color

only uniqueness constraint, for simplicity

$V(M_1^*) = 2.7$ unstable

$V(M_2^*) = 2.7$ stable with margin 0.1

- Two solutions of minimum energy 2.7
- M_1^* is unstable: a small perturbation of the costs causes M_1^* change to M_2^*: eg. $0.1 \mapsto 0.1 - \varepsilon$, $\varepsilon > 0$
- **Absorbance:** M is absorbing if every rejected element $r \notin M$ has at least one accepted element in $X(r) \cap M$ of *strictly smaller cost* than r
- M_1^* is not absorbing because of $c(3,3) = 0.1$
- M_2^* is absorbing with the *margin* of 0.1

  Even if any subset of unmatched elements decrease their cost by 0.1, the solution M_2^* is still absorbing
- Can we find a stable solution given a margin value? Yes. And it is unique.

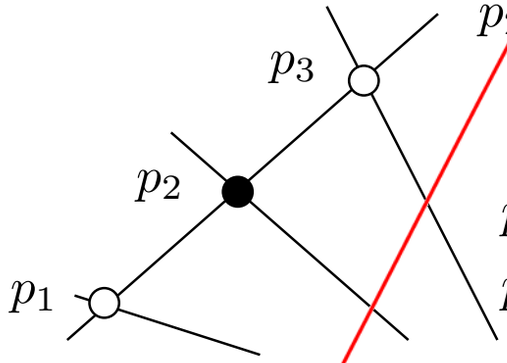
Confidently Stable Matching

Given:

1. matching participants $i \in I$, $j \in J$ — left image, right image pixels
2. putative correspondences $p \in T$, $p = (i, j)$, $i \in I$, $j \in J$ — matching table entries
3. symmetric occlusion E relation on pairs p, $q \in T$
4. interval statistic for image ~~correlation~~ dissimilarity $c(p) = \left[\underline{c}(p), \overline{c}(p)\right]$ — interval representing uncertainty

Construct:

1. simple 'occlusion' graph $\mathcal{G} = (T, E)$



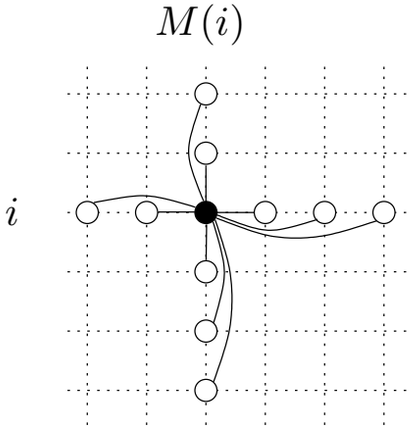
p_2 accepted:
p_1 transparent
p_3 invisible in C_1

$p_1 \in M \Rightarrow p_2 \notin M$

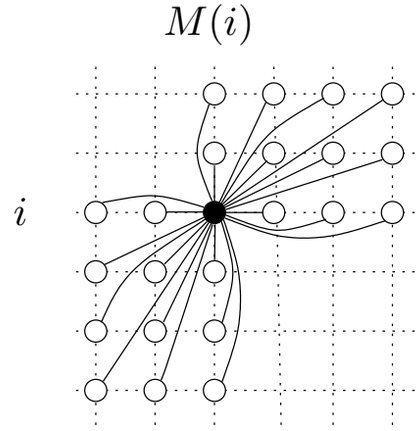
$p_2 \in M \Rightarrow p_1 \notin M$

$p = (i, M(i))$ occludes/rejects neighbors in $\mathcal{G}$

Obs: Independent vertex sets of $\mathcal{G}$ are matchings.



uniqueness



uniqueness + ordering

2. graph orientation by intervals c:

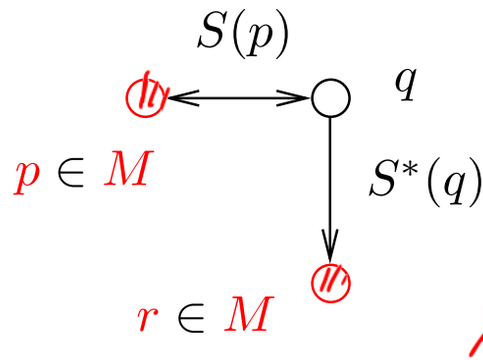
if $c(p) \cap c(q) \neq \emptyset$ then oriented bidirectionally $p \bigcirc\!\!\longleftrightarrow\!\!\bigcirc q$ none is better

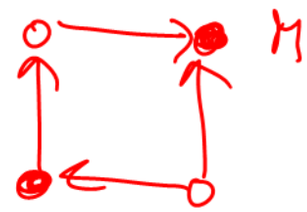
if $c(q) \not\geq c(p)$ then oriented unidirectionally $p \bigcirc\!\!\longrightarrow\!\!\bigcirc q$ arrow towards better

Strict Sub-Kernel (SSK) of an Oriented Graph

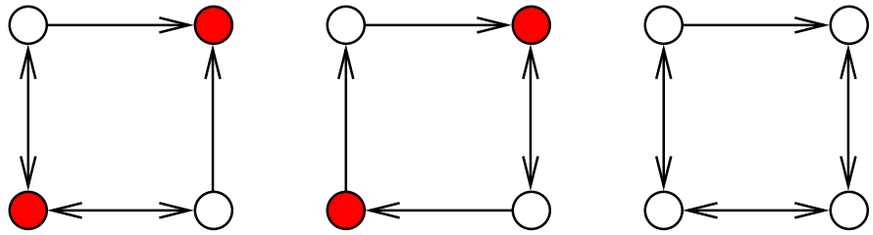
Def. *Independent vertex subset M is a strict sub-kernel of $\mathcal{G}$ if for every $p \in M$ there is a $r \in M$ such that $r \in S^*(q)$ for every $q \in S(p)$.*

"every successor of M is absorbed by M"





Examples



single SSK | single SSK | no SSK | no SSK | 2 SSK but not interval orientation

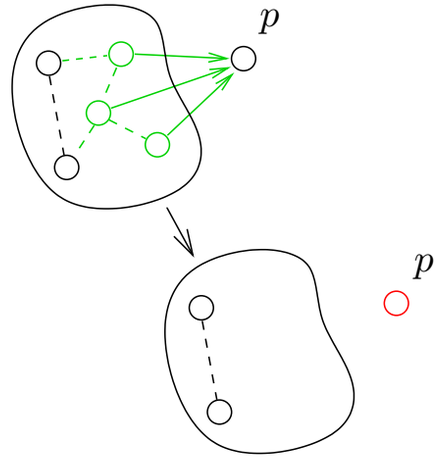
Proposition: Let $\mathcal{G}^*$ be graph $\mathcal{G}$ oriented by the interval overlap relation. Then (1) there is a unique maximum strict sub-kernel (SSK) of $\mathcal{G}^*$ for any graph $\mathcal{G}$ and (2) it can be found in polynomial time.

Proposition: SSKs are absorbing, hence stable.

A Simple Algorithm

Sink Stripping Algorithm

1. Initialize $M := \emptyset$.
2. Find a sink $s \in V(\mathcal{G}^*)$. a sink has no successor
3. If there is no sink in $\mathcal{G}^*$, terminate and return M.
4. Add s to M.
5. Remove s and all its predecessors $P(s)$ from $\mathcal{G}^*$.
6. Go to Step 2.



$O(N^3)$??

Notes

- Stereo matching with sink-stripping alg has complexity $O(N^5)$, an $O(N^3 \log N)$ algorithm exists, hints in: [Šára, ECCV 2002]
- Related to Stable Marriage Problem [von Neumann 1944, Gale and Shapley, AMM 1962]

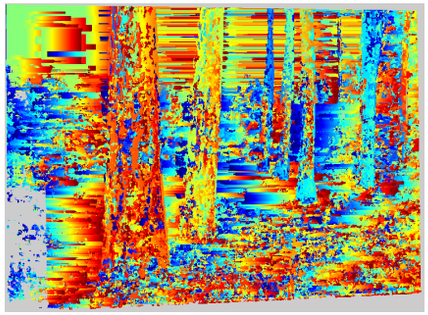
Algorithm Comparison

MAP with Min-Cost Path (DP)

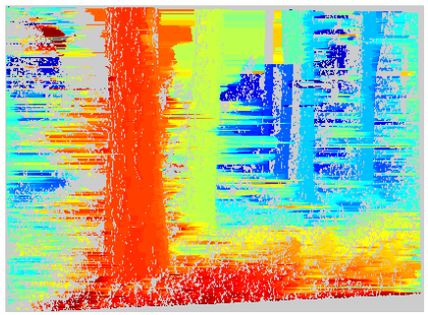
- semi-dense disparity map
- 'illusions' if ordering does not hold
- $O(N^3)$ algorithm

Strict Sub-Kernel (SSK)

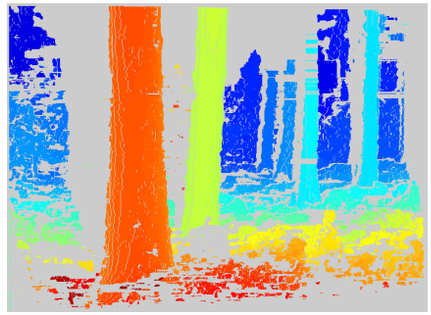
- semi-dense disparity map
- 'holes' if ordering does not hold
- $O(N^3 \log N)$ algorithm



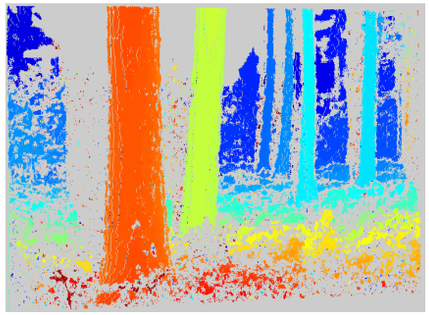
WTA



DP

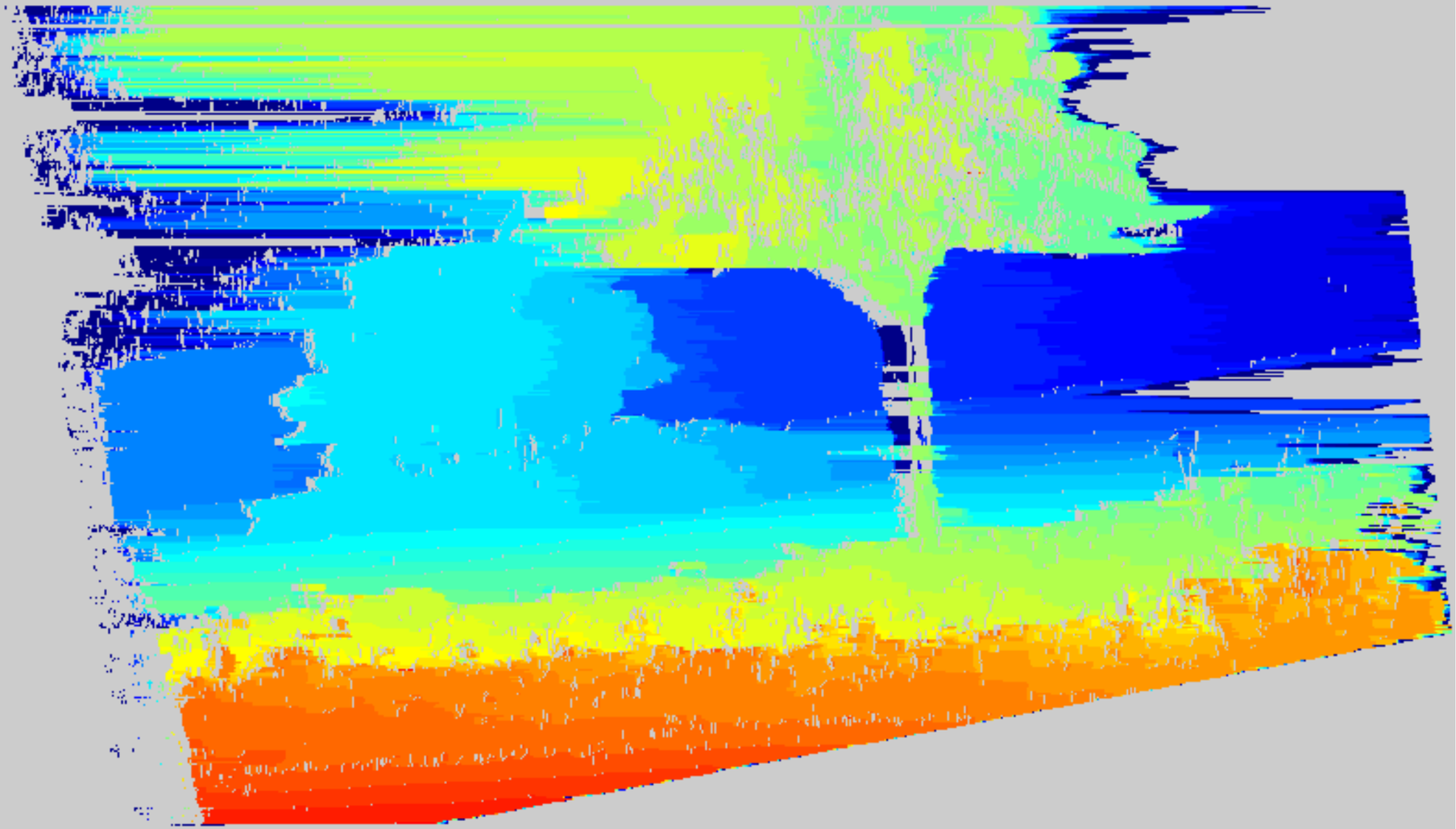
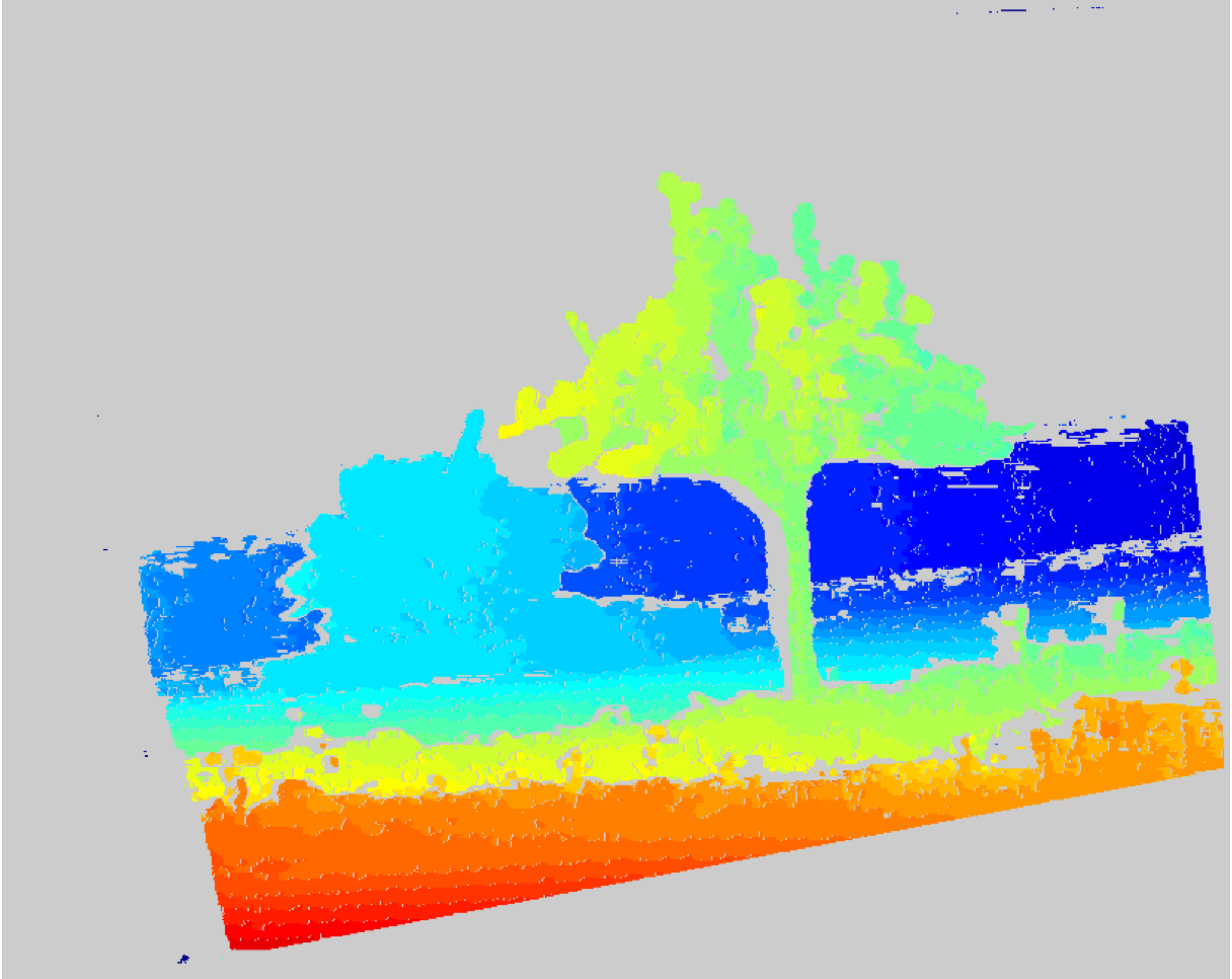
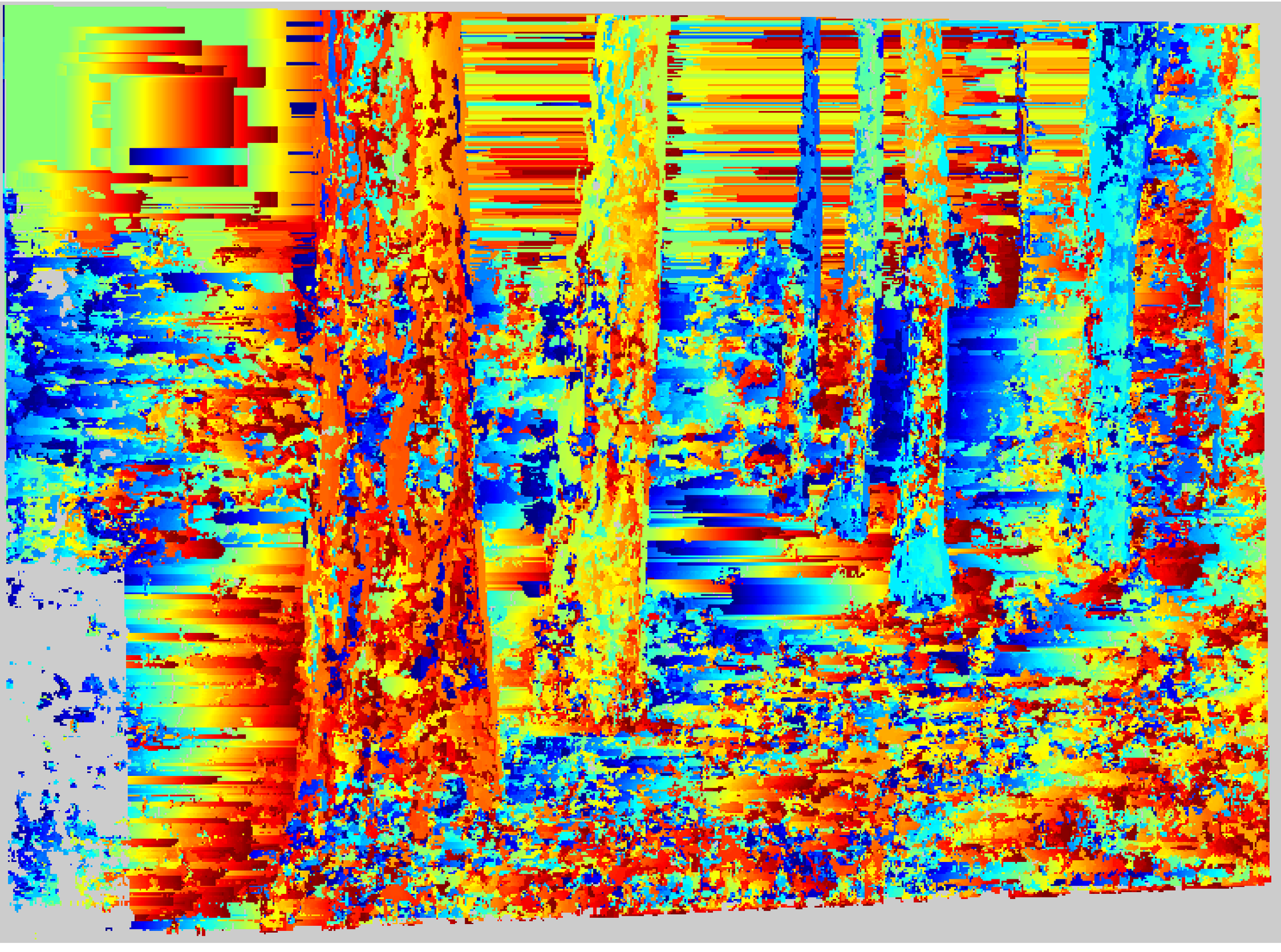
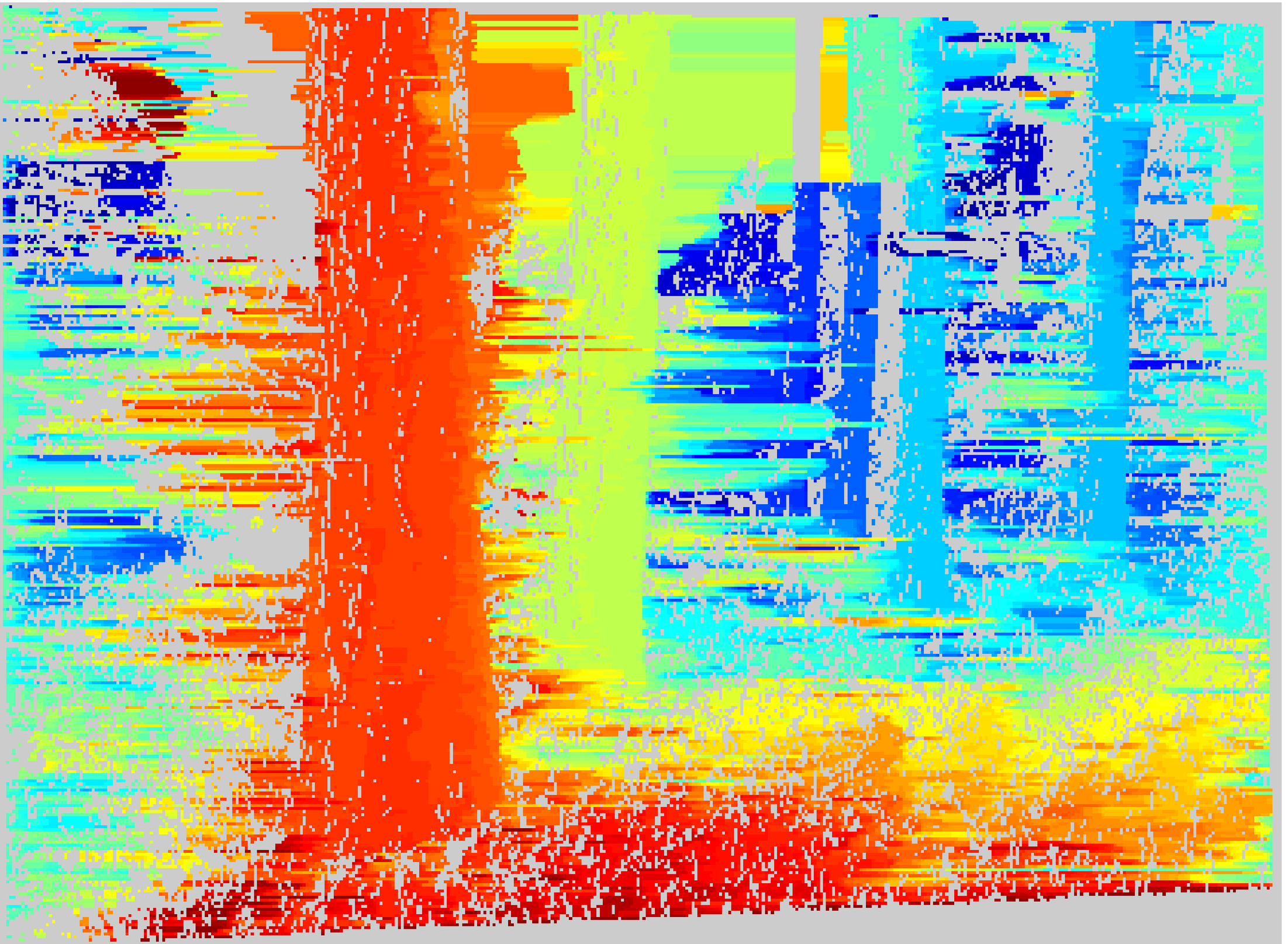
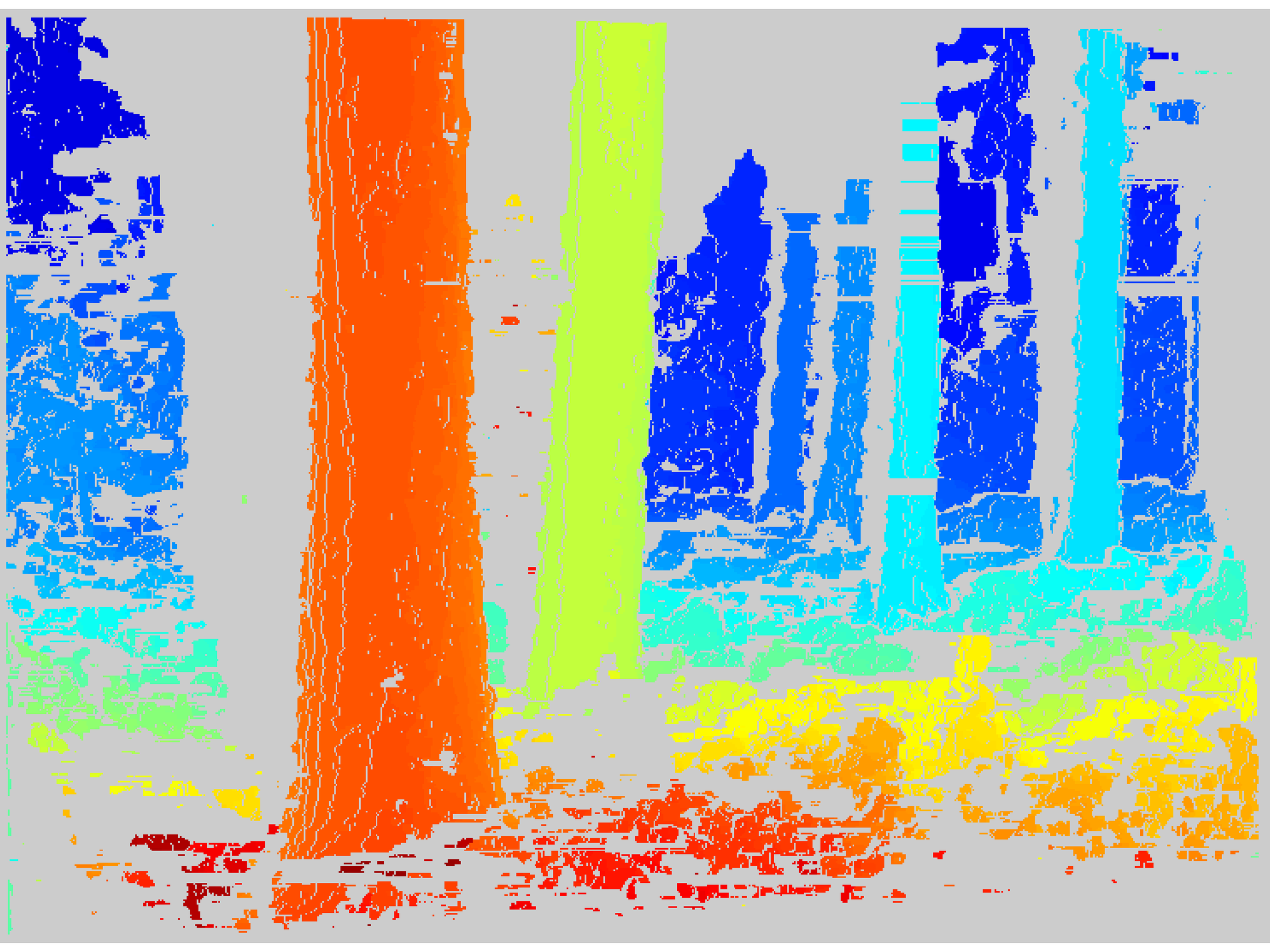
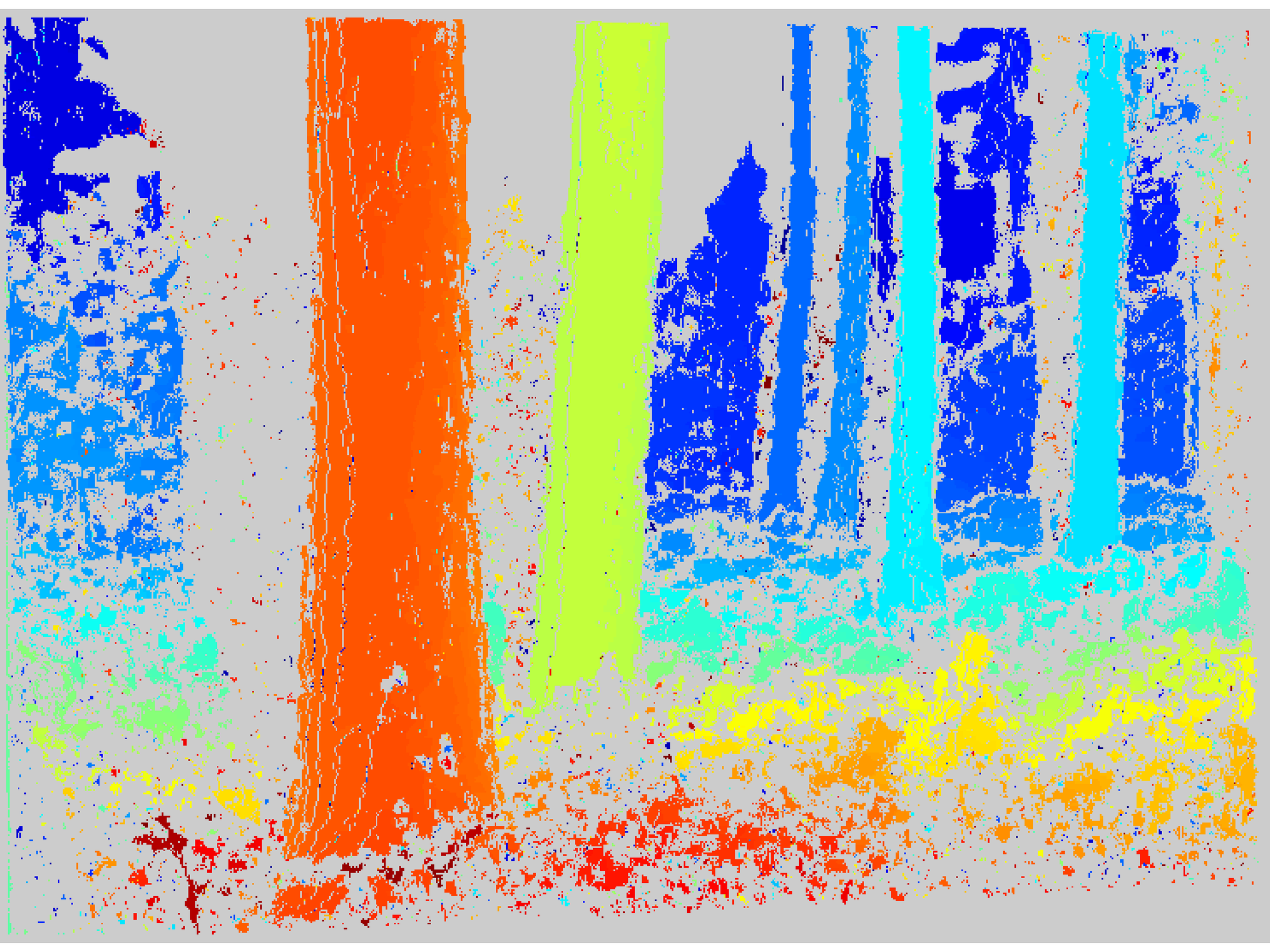


SSK (~~occl.~~) ordered



SSK (w/o occl.)

Notes

- WTA: Simple things do not work
- SSK: Stronger model vs. model invalidity

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