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In vivo Conductivity Estimation with Symmetric Boundary Elements

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***Abstract*— A symmetric boundary element method, whose high accuracy for the forward electroencephalography (EEG) problem has recently been reported, is applied to *Electrical Impedance Tomography* (EIT), i.e. the estimation of conductivity in vivo, using current injection on the scalp. Current injection is very easy to model, as the current flow is an explicit unknown in the symmetric BEM. The EEG and EIT have the same system matrices, and no extra assembly is required. A gradient descent method allows to estimate the conductivities, by minimizing the discrepancy between simulated and measured potentials. A variant is proposed for simultaneous estimation of the current injection intensity. We demonstrate the ability of the method to recover the skull-scalp conductivity ratio, on simulations and on measured data.**

***Keywords*— Boundary Element Method (BEM), Electrical impedance tomography (EIT), integral method, Electroencephalography (EEG)**

I. Introduction

The inverse electroencephalography problem is quite sensitive to the precise knowledge of the conductivity distribution inside the head [1]. The skull in particular requires much attention because of its low conductivity, its non-homogeneous character and its complex geometry. *In vitro* conductivity measurements have demonstrated the high variability of skull and white matter conductivity between subjects. It is therefore important to develop non-invasive methods for routine *in vivo* conductivity measurement.

The extreme sensitivity of the electric potential to the conductivity profile of the head, and particularly the skull, can actually be turned into an advantage, in the case where some electrical source parameters can be controlled: this is the gist of Electrical Impedance Tomography. Variants in this domain concern the ways in which the electrical source is controlled.

One approach consists of using source localization from magnetoencephalography somatosensory evoked fields to constrain an EEG forward problem. Conductivity values for a given head model are estimated by matching simulated potentials to recorded potentials [2–4]. Injected-current EIT, which we focus on in this study, imposes a low-intensity current on the scalp through selected EEG electrodes, and presents the advantage of using an inexpensive experimental setup readily accessible in all neurophysiology units [5].

Our method for EIT follows the general lines of [6], but it is based on the symmetric BEM, which shows high accuracy for the forward EEG, thus requiring triangulations with less elements to reach a prescribed precision [7, 8]. In the symmetric BEM, both the potential and the flux are explicit unknowns, allowing the scalp current to be imposed very easily, and the forward matrix for EIT is exactly the same as for EEG. We propose an algorithm which minimizes the discrepancy between simulated and measured potentials by a gradient descent on the conductivity values. The method is applied to a real EIT dataset, and in spite of the lack of injection current measurements, it appears successful in recovering the skull-to-scalp conductivity ratio. The estimated ratio is of order 1/25, i.e. in agreement with recent results in the literature.

II. Symmetric BEM

A. A common model for EEG and EIT

Consider an electric potential V which satisfies, inside the head volume, a Poisson equation [9]

$$\nabla \cdot (\sigma \nabla V) = f \quad \text{in } \Omega \tag{1}$$

with the following boundary condition on the scalp:

$$\sigma \partial_{\mathbf{n}} V = j \quad \text{on } \partial\Omega \ . \tag{2}$$

In an EEG model, the sources inside the brain are represented by f, and the current on the scalp, j, vanishes because of the non-conducting medium (air) around the head. Conversely, in our EIT model, sources inside the head are assumed negligible ($f = 0$) and the current on the scalp j is imposed.

In this paper we consider a three-layer head model. Conductivities of the brain+CSF[1], the skull, and the scalp are respectively denoted σ_1, σ_2 and σ_3. The surfaces enclosing these homogeneous conductivity regions are denoted S_1 (inner skull boundary), S_2 (skull-scalp interface) and S_3 (scalp-air interface).

B. Symmetric BEM formulation

A careful application of the Green representation theorem leads to a linear relationship through integral operators between the potential V and the normal current $\sigma \partial_{\mathbf{n}} V$ on each of the surfaces[2], driven by a source term involving the inner current sources f and/or the injected current j. More precisely, let V_i, resp. $p_i = (\sigma \partial_{\mathbf{n}} V)_i$ denote the restriction to surface S_i of the potential, resp. the normal current. With the formalism presented in [8] the system (1)-(2) for a three-layer head model becomes:

$$\mathsf{A}_\sigma \, [V_1 \quad V_2 \quad V_3 \quad p_1 \quad p_2]^T = \mathsf{B}_\sigma \tag{3}$$

[1] cerebro-spinal fluid
[2] denoting $\mathbf{n}$ the outward-pointing normal. Unlike $\partial_{\mathbf{n}} V$, the quantity $\sigma \partial_{\mathbf{n}} V$ is continuous and it is legitimate to consider its restriction to a surface of discontinuity of conductivity.

where A_σ is a matrix and B_σ a vector which are now detailed. The symmetric matrix A_σ has a block structure

$$\mathsf{A}_\sigma = \begin{pmatrix} \mathsf{N}_\sigma & \mathsf{D}^T \\ \mathsf{D} & \mathsf{S}_\sigma \end{pmatrix} \tag{4}$$

with

$$\mathsf{N}_\sigma = \begin{bmatrix} (\sigma_1+\sigma_2)\mathsf{N}_{11} & -\sigma_2\mathsf{N}_{12} & 0 \\ -\sigma_2\mathsf{N}_{21} & (\sigma_2+\sigma_3)\mathsf{N}_{22} & -\sigma_3\mathsf{N}_{23} \\ 0 & -\sigma_3\mathsf{N}_{32} & \sigma_3\mathsf{N}_{33} \end{bmatrix}$$

$$\mathsf{S}_\sigma = \begin{bmatrix} (\sigma_1^{-1}+\sigma_2^{-1})\mathsf{S}_{11} & \sigma_2^{-1}\mathsf{S}_{12} \\ \sigma_2^{-1}\mathsf{S}_{21} & (\sigma_2^{-1}+\sigma_3^{-1})\mathsf{S}_{22} \end{bmatrix}$$

$$\mathsf{D} = \begin{bmatrix} -2\mathsf{D}_{11} & \mathsf{D}_{12} & 0 \\ \mathsf{D}_{21} & -2\mathsf{D}_{22} & 0 \end{bmatrix}$$

The blocks N_{ij},S_{ij} and D_{ij} only depend on the geometric structure of the meshes, and not on the conductivities. The blocks N_{ij} and D_{ij} map a potential V_j on S_j to a quantity defined on S_i. The blocks S_{ij} map a normal current p_j on S_j to a quantity defined on S_i.

The vector B_σ on the right-hand side of (3) was derived in [8] for the EEG case: supposing f to be supported in the brain compartment, and j in (2) to vanish on the scalp, then

$$\mathsf{B}_\sigma = [(\sigma_1\partial_{\mathbf{n}}v)_1 \quad 0 \quad 0 \quad (v)_1 \quad 0]^T \tag{5}$$

where v is the solution of $\sigma_1\Delta v = f$, i.e. a potential driven by the same source term, but in an infinite domain with conductivity σ_1.

To apply the symmetric BEM to EIT, we generalize the right-hand side term to the case of injected current on the scalp. In the case where internal sources f vanish and $j \neq 0$,

$$\mathsf{B}_\sigma = \left[0 \quad -\mathsf{D}_{23}^T j \quad \mathsf{D}_{33}^T j - \frac{j}{2} \quad 0 \quad \sigma_3^{-1}\mathsf{S}_{23} j\right]^T . \tag{6}$$

Details on the derivation of the above term, using arguments similar to those of [8], will be detailed in a forthcoming article. If both internal sources f and injected current j are present, the right-hand side terms (5) and (6) are simply summed.

C. Discretization with P1-P0 elements

The vector of unknowns $[V_1 \quad V_2 \quad V_3 \quad p_1 \quad p_2]^T$, once discretized, combines the $P1$ coefficients of the potential V on all vertices of surface S_1, S_2 and S_3, and the $P0$ coefficients of the flux $p = \sigma\partial_{\mathbf{n}}V$ on surfaces S_1 and S_2. The injection current j is discretized with $P0$ boundary elements on the scalp, i.e. a constant value on the two triangles representing the injection and extraction electrodes and zero elsewhere.

D. Validation

We introduce two potentials: V_f the solution of an EEG forward problem (1)-(2) with $f = \nabla\cdot\vec{J}_p$ and $j = 0$, and V_j the solution of an EIT forward problem (1)-(2) with $f = 0$ and $j = \delta_{r_i} - \delta_{r_e}$. Then from the principle of reciprocity [10]

$$\int_\Omega \vec{J}_p(r)\cdot\nabla V_j = V_f(r_i) - V_f(r_e) \ .$$

The electric field of EIT ∇V_j can be viewed as a *lead field* for EEG. We have used this observation to validate the EIT solution V_j, for the case of a dipolar EEG source $\vec{J}_p(r) = \vec{q}\,\delta_{r_0}$; first on concentric spheres, with an analytical EEG solution, and then also for a realistic head geometry model, with the forward EEG solution V_f calculated with the symmetric BEM. Relative errors were very small (of the order of 1%).

III. Inverse EIT Solution

A series of M EIT experiments was performed on the same human subject. In each experiment $m = 1,\ldots,M$, we noted

- the electrode labels I_m used for current injection,
- the potential v_m measured on the remaining electrodes.

The potential could not be reliably measured on the injection and extraction electrodes because of the impedances involved.

A. Case of current injection of known intensity

We assume the intensity of the current injection λ_m to be known. The scalp current injection pattern is

$$j_m(r) = \lambda_m\, u_m(r) \tag{7}$$

where u_m is of the form $\delta(r_1)-\delta(r_2)$, with unit intensity. For a fixed conductivity distribution, the forward EIT problem can be solved using (3)-(6), yielding a potential $V(\sigma, j_m)$ on the scalp. We seek the conductivity distribution $\sigma = (\sigma_1,\sigma_2,\sigma_3)$ which minimizes the cost function

$$E(\sigma) = \frac{1}{2}\sum_{m=1}^{M}\|V(\sigma,j_m) - v_m\|^2_{\text{meas}} \ . \tag{8}$$

The norm $\|\ \|^2_{\text{meas}}$ is a discrete ℓ^2 norm over the measurement electrodes: denoting r_k the position of electrode k,

$$\|V - v_m\|^2_{\text{meas}} = \sum_{k\notin I_m}|V(r_k) - v_m(r_k)|^2 \ .$$

The gradient of the criterion (8) is equal to

$$\nabla_\sigma E(\sigma) = \sum_{m=1}^{M}\langle V(\sigma,j_m) - v_m,\ \nabla_\sigma V(\sigma,j_m)\rangle_{\text{meas}} \tag{9}$$

where

$$\langle V_1,\, V_2\rangle_{\text{meas}} = \sum_{k\notin I_m} V_1(r_k)\, V_2(r_k) \ .$$

In our piecewise-constant conductivity model,

$$\nabla_\sigma V = (\partial_{\sigma_1}V,\, \partial_{\sigma_2}V,\, \partial_{\sigma_3}V) \ .$$

Differentiating (3) with respect to σ_i shows that these partial derivatives satisfy

$$\begin{aligned}\mathsf{A}_\sigma\,[\partial_{\sigma_i}V_1 \quad \partial_{\sigma_i}V_2 \quad \partial_{\sigma_i}V_3 \quad \partial_{\sigma_i}p_1 \quad \partial_{\sigma_i}p_2]^T = \\ -\,\partial_{\sigma_i}\mathsf{A}_\sigma\,[V_1 \quad V_2 \quad V_3 \quad p_1 \quad p_2]^T + \partial_{\sigma_i}\mathsf{B}_\sigma \quad \forall i\end{aligned} \tag{10}$$

Given the form of A_σ in (4), and decomposing (with a straightforward notation) $\mathsf{N}_\sigma = \sum\sigma_i\mathsf{N}_i$ and $\mathsf{S}_\sigma = \sum\sigma_i^{-1}\mathsf{S}_i$, it is clear that

$$\partial_{\sigma_i}\mathsf{A}_\sigma = \begin{pmatrix} \mathsf{N}_i & 0 \\ 0 & -\sigma_i^{-2}\mathsf{S}_i \end{pmatrix} \ .$$

Any multidimensional method can be used to find $\sigma = (\sigma_1, \sigma_2, \sigma_3)$ minimizing E. We have chosen a gradient descent algorithm (Algorithm 1).

Algorithm 1 Iterative gradient descent for σ

```
Initialize σ
repeat
  for each experiment m = 1, ..., M do
    compute V(σ, j_m) solution of (3)-(6)
    for i = 1, 2, 3 do
      compute ∂σ_i V solution of (10)
    end for
  end for
  compute ∇_σ E with (9)
  perform a line search to find α minimizing E(σ − α∇_σ E)
  σ ← σ − α∇_σ E
until ‖∇E(σ)‖ < threshold
```

B. Case of unknown current injection intensity

Depending on the experimental setup, the injected current amplitude may not be known and may not even be constant during the experiment. For this reason, we propose an alternative inverse EIT algorithm in which the current intensities $\{\lambda_m\}$ in (7) are also estimated. In this case, it is not possible to recover the scalp conductivity, which will therefore be set to a fixed value. We use the modified cost function

$$E'(\sigma) = \min_{\{\lambda_m\}} \frac{1}{2} \sum_{m=1}^{M} \|V(\sigma, \lambda_m u_m) - v_m\|^2_{\text{meas}} . \qquad (11)$$

The minimizing λ_m can be computed explicitly due to the linearity of V with respect to source intensities

$$\hat{\lambda}_m(\sigma) = \frac{\langle V(\sigma, u_m), v_m\rangle_{\text{meas}}}{\langle V(\sigma, u_m), V(\sigma, u_m)\rangle_{\text{meas}}} \qquad (12)$$

and therefore

$$E'(\sigma) = \frac{1}{2} \sum_{m=1}^{M} \left\| \hat{\lambda}_m(\sigma)\, V(\sigma, u_m) - v_m \right\|^2_{\text{meas}} .$$

Algorithm 1 remains valid, if the gradient (9) is replaced by

$$\nabla_\sigma E'(\sigma) = \sum_{m=1}^{M} \langle \hat{\lambda}_m(\sigma) V(\sigma, u_m) - v_m, \nabla_\sigma \left(\hat{\lambda}_m(\sigma)\, V(\sigma, u_m) \right) \rangle_{\text{meas}}$$

IV. Experimental results

A. Experimental setup

A 32-channel EEG helmet was positioned on the subject's head, and 20 injection-extraction electrode pairs selected. A current was applied to each of these electrode pairs successively. We used a series of square pulses with a duration of 0.5 ms, an amplitude of about 0.1 mA and a repetition frequency of 110 Hz. The potential was sampled at 10 kHz and averaged over more than 1000 selected, artifact-free repetitions. For each electrode stimulation pair, the measured EIT potential v_m was defined as the peak value of the averaged potential, the peak occurring simultaneously at all measurement electrodes. The injection current was not measured during acquisition, and therefore we used the estimation procedure outlined in Section III-B. Consequently, it was not possible to estimate the absolute tissue conductivities, but only their ratio to the scalp conductivity.

B. Head model and system assembly

A magnetic resonance (MR) image of the subject's head was segmented into three surfaces using ASA (Advanced Source Analysis, ANT). The meshes generated were of sufficient quality for BEM and did not require any post-processing. The number of vertices per surface was 510 (brain-skull), 510 (skull-scalp) and 1222 (scalp-air). For a fixed conductivity distribution, the EIT system matrix A_σ, as well as the source term B_σ were assembled, requiring approximately 8 and 5 minutes, respectively, on a Pentium PC with 3.4 GHz CPU and 2 GB of RAM. Updating these matrices for different σ is straightforward given (4) and (6).

C. Conductivity estimation

We tested the algorithm on simulated data, generated with the realistic three-layer head model described in Section IV-B, for conductivity values $\sigma_{\text{brain}} = 1.2, \sigma_{\text{skull}} = 0.0308$, the conductivity of scalp being fixed to $\sigma_{\text{scalp}} = 1$. Results are presented in Table I. Estimating σ_{skull} and σ_{brain} simultaneously, we noticed that the estimation method was not sensitive enough to brain conductivity: the gradient $\partial_{\sigma_{\text{brain}}} E(\sigma)$ was much smaller than $\partial_{\sigma_{\text{skull}}} E(\sigma)$, and the brain conductivity almost did not change when optimizing over both parameters.

Optimizing only over the skull conductivity, we tested the variability of the estimated skull conductivity with respect to the fixed brain conductivity. A change of up to 50 % in σ_{brain} resulted in a change of less than 16 % in σ_{skull} for the simulated dataset.

Table II presents conductivity estimation results on the real dataset, optimizing only on the skull conductivity. The skull-scalp conductivity ratio recovered is 25, which is consistent with recent results in the literature [2–6].

The normalized cost function $E(\sigma)/\sum_{m=1,\ldots,M} \|v_m\|^2_{\text{meas}}$ is an indication of the goodness of fit of the model. For the real dataset, Figure 1 (top) shows the normalized cost according to the skull-scalp conductivity ratio, with brain and scalp conductivities set equal to one. The full curve shows the normalized cost (11) when the current intensities are estimated as well as the conductivities. The dashed curve corresponds the normalized cost (8) for fixed current intensity values, set to $\hat{\lambda}_m(\sigma_{\text{opt}})$, where σ_{opt} is the conductivity value minimizing the full curve. A zoom on the minimizing region in Figure 1 (bottom) shows that the cost displays a clear minimum even in the case of estimated current (full curve). It is also evident that the knowledge of current injection (e.g. through its measurement) makes the minimum of the cost function much sharper.

TABLE I

SIMULATED DATA, FOR $\sigma_{\text{BRAIN}} = 1.2$, $\sigma_{\text{SKULL}} = 0.0308$, $\sigma_{\text{SCALP}} = 1$.

fixed conductivities	initialization	estimated conductivities	time elapsed
$\sigma_{\text{scalp}} = 1$	$\sigma_{\text{skull}} = 0.1$ $\sigma_{\text{brain}} = 1.12$	$\sigma_{\text{skull}} = 0.0317$ $\sigma_{\text{brain}} = 1.12$	648 s
$\sigma_{\text{scalp}} = 1$ $\sigma_{\text{brain}} = 1.2$	$\sigma_{\text{skull}} = 0.1$	$\sigma_{\text{skull}} = 0.0309$	543 s
$\sigma_{\text{scalp}} = 1$ $\sigma_{\text{brain}} = 1.8$	$\sigma_{\text{skull}} = 0.1$	$\sigma_{\text{skull}} = 0.0259$	669 s

TABLE II

REAL DATA

fixed conductivities	initialization	estimated conductivities	time elapsed
$\sigma_{\text{scalp}} = 1$ $\sigma_{\text{brain}} = 1$	$\sigma_{\text{skull}} = 0.02$	$\sigma_{\text{skull}} = 0.0408$	485 s
$\sigma_{\text{scalp}} = 1$ $\sigma_{\text{brain}} = 1$	$\sigma_{\text{skull}} = 0.1$	$\sigma_{\text{skull}} = 0.0407$	597 s
$\sigma_{\text{scalp}} = 1$ $\sigma_{\text{brain}} = 1.2$	$\sigma_{\text{skull}} = 0.1$	$\sigma_{\text{skull}} = 0.0394$	594 s

V. CONCLUSION

We have presented a novel conductivity estimation procedure with the following main features:

- It uses the symmetric BEM formulation, which guarantees a very accurate realistic forward model.
- The forward matrix of EIT is the same as the one of EEG, and it is very easy to update for different conductivity profiles.
- Current injection at the electrodes is very simple to model, since the current flow is an explicit unknown in the symmetric BEM.
- The inverse EIT problem is formulated as a minimization of a cost function, whose gradient can be computed.
- The optimization based on gradient descent is fast: the algorithm converges in less than 12 minutes.
- The method does not require injection current intensity to be measured.

Results on simulated and on real datasets show the pertinence of the method for the estimation of the skull-scalp conductivity ratio. Further validation on more subjects is underway. According to our preliminary results, scalp injected EIT does not appear reliable for estimating brain conductivity. Alternative methods in which the current source is located inside the brain, such as somatosensory evoked potentials, are probably a better choice for brain conductivity estimation.

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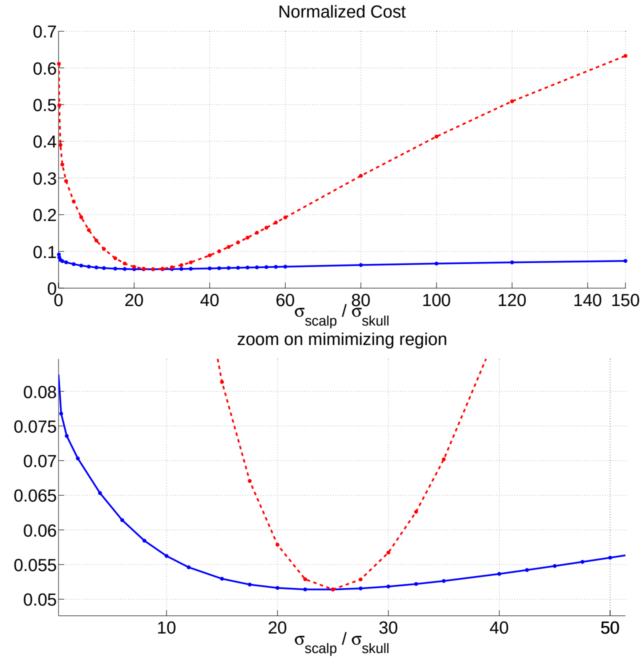

Fig. 1. Normalized cost for the real data (top). Zoom on the minimizing region (bottom). Full line: with simultaneous estimation of injected current. Dashed line: with fixed injected current (see text).

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