Accurate Boundary Element Method for the Electro- and Magnetoencephalography Forward Problem

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The forward E/MEG problem consists of finding a surface potential from the Poisson equation, given the sources and the head geometry. We describe a novel symmetric boundary element method (BEM) for solving the forward problem using a realistic meshed surface head model with generalized topology. We found that this new method is more accurate and less computationally expensive than the classical double layer method.

1 Introduction

Electroencephalography (EEG) [1] and magnetoencephalography (MEG) [2] are non-invasive diagnostic and observational techniques based on measuring the electric and magnetic field on the scalp, linked to the neural activity.

Better techniques are needed to retrieve the spatial information from the M/EEG data, as most of the analysis is still done by projecting the measurements onto the head surface, or by approximating the activated zone by a single dipole.

In this contribution we will concentrate on the *forward problem*, which consists of calculating the surface electric field given the sources inside the head. Its accurate solution is a necessary prerequisite for solving the reconstruction problem (also called the *inverse problem*).

2 Methods

The forward EEG problem involves finding a potential V from the Poisson equation $\nabla \cdot (\sigma \nabla V) = \nabla \cdot \mathbf{J}^p$, where $\mathbf{J}^p$ are the known primary current sources. The magnetic field $\mathbf{B}$ is then found by simple integration.

Under the piecewise conductivity assumptions, the Poisson partial differential equation can be transformed into a set of Laplace equations $\sigma \Delta V = f$, connected with boundary conditions. These can in turn be converted into the integral form using the Green identities, and discretized. This approach, called the Boundary Element Method, is now a classical way of solving the forward problem [3, 4]. Its main advantage with respect to the finite difference method (FDM) or the finite element method (FEM) resides in the fact that it only uses values on surfaces as unknowns and the dimensionality of the problem and the number of unknowns is thus greatly reduced.

All known BEM implementations for the M/EEG problem are based on the so-called *double layer* integral formulation [3]. However, these methods are unacceptably inaccurate for sources close to one of the surfaces (discontinuities of the conductivity) [4]. An example of this behavior can be seen in Fig. 1, top left. It turns out that there are other integral formulations possible, for example the dual, *single layer* formulation. The main contribution of our work is then the introduction of a *symmetric method* [5], combining the single and double layer methods. It uses as unknowns both potential V and the current flow $p = \sigma \nabla V$, and yields a symmetric system matrix. This method, new in the M/EEG field, turns out to be much more precise than the alternatives for our application. Our second contribution consist of releasing the requirement to model the head as a set of nested volumes; more general configuration is now possible.

We have carefully discretized the symmetric method using the Galerkin approach to preserve its accuracy. We have implemented a parallel solver that is currently capable of treating meshes with up to 30000 triangles.

3 Results

We have tested our method with synthetic spherical head models for which the analytical solution is available. We found the symmetric method to perform better then the alternatives in the multilayer head model case (see Fig. 1, top left). The accuracy of the new method is preserved for much longer as the dipole approaches the boundary. Also the new method is much less affected by the high ratio of tissue conductivities (Fig. 1,top right).

We have also tested the symmetric method on a realistically shaped four layer head model with about 13000 points and 26000 triangles (Fig. 1, bottom), generated by segmentation from MRI head scan data. Surprisingly, it turned out that the new method was computationally less expensive than the alternative methods, when equivalent discretization was used, in spite of using more variables. This is because its system matrix is somewhat redundant thanks to the symmetry and the possibility of reusing a large part of the calculations.

4 Conclusions

We have shown that the classical integral formulation used for EEG and MEG calculations by the BEM is not unique. We have presented an alternative approach, appealing by its symmetry and its superior accuracy in semi-realistic geometry. This brings the project of accurate electromagnetic simulation of the human brain much closer to the limits of available technology. Nevertheless, advanced acceleration techniques will have to be used before it becomes viable.

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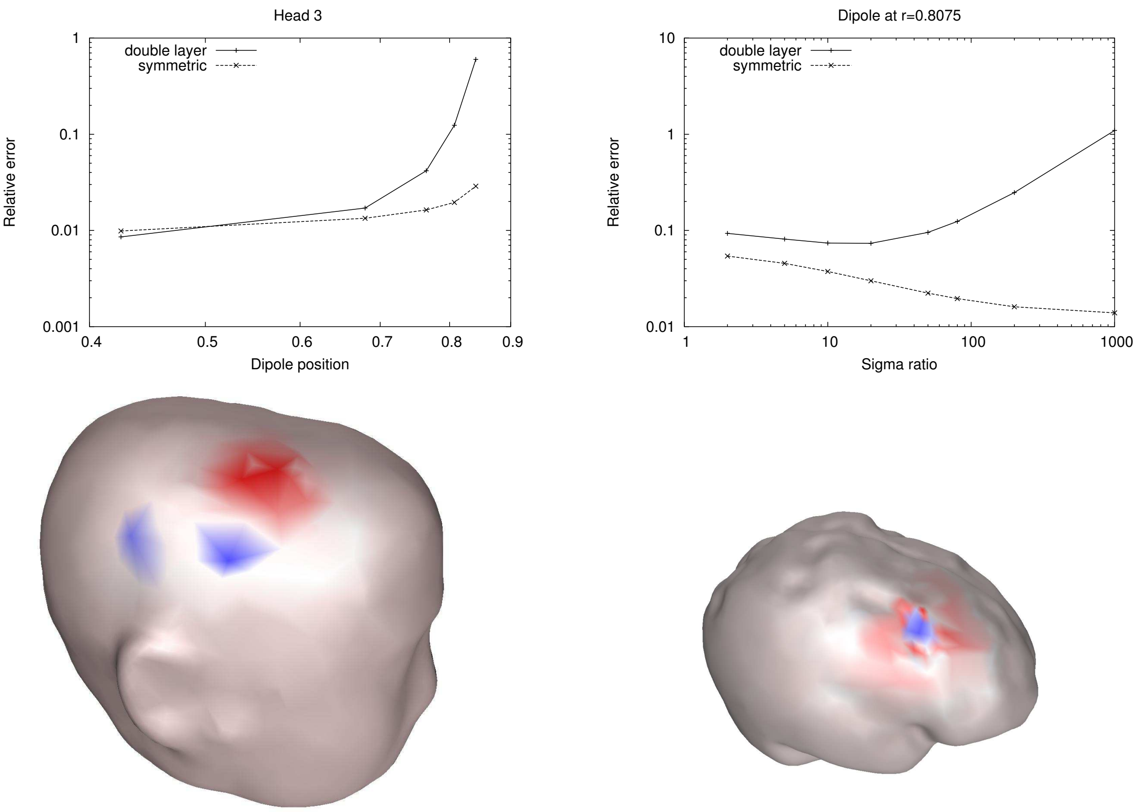

Fig. 1: The relative error versus dipole position *(top left)* for the classical double layer method and the new symmetric method. The relative error versus the ratio of conductivities *(top right)* for the same two methods. The calculated field on the surface of the scalp *(bottom left)* and on the cortex *(bottom right)*, for a realistic head model and a dipole close to the cortex.

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