Classes of logics representable as kernels of measures

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Abstract

We investigate the kernels of (group-valued) measures on Boolean algebras. They form a special class of orthomodular posets — the class of *kernel logics*. We show that every poset can be embedded into a kernel logic. This result generalizes the construction developed for lattices by Kalmbach [5]. We establish other sufficient conditions for an orthomodular poset to be a kernel logic. As a consequence, we find out that every group is the automorphism group of a kernel logic.

1 Introduction

The classical Kolmogorov model of probability is based on the notion of Boolean algebra. It assumes that every pair of events is simultaneously observable. This principle is violated in some applications of probability, e. g. in quantum mechanics, artificial intelligence, sociology, etc. Therefore several more general structures have been suggested as the event structure of such systems (see e. g. [1, 6, 12]). Among them, orthomodular posets and orthomodular lattices play an important role. However, these structures appeared to be too general for the development of an applicable generalized probability theory. For example, the additivity proofs [9, 13] are very complex and some important theorems even do not have an analogue in the generalized model. Therefore, kernel logics have been suggested as an event structure [10, 11]. They are close to Boolean algebras and described fully in Boolean terms. Still they allow to describe systems in which simultaneous observability is not assumed.

Here we present some observations on the algebraic properties of kernel logics. For the basic facts about orthomodular posets and lattices, we refer to [6, 12], but the knowledge of special "orthostructure" techniques, e. g. horizontal sums and Greechie diagrams, is not necessary for substantial part of the paper. For more details on kernel logics, we refer to [10, 11].

In this paper, we shall deal only with set-representable (=concrete) orthomodular posets. Let X be a nonempty set and let $\mathcal{L} \subset 2^X$ such that $X \in \mathcal{L}$ and $B \setminus A \in \mathcal{L}$ whenever $A, B \in \mathcal{L}$, $A \subset B$. Then $\mathcal{L}$ is called a *class of subsets* of X [1]. A class of subsets of a set X becomes an orthomodular poset if we take the inclusion for the ordering and define the orthocomplementation as $A \mapsto X \setminus A$. A *concrete logic* is an orthomodular poset isomorphic to a class of subsets [12].

Let $\mathcal{L}$ be an orthomodular poset, G a commutative group. A *G-valued measure* on $\mathcal{L}$ is an additive mapping $m : \mathcal{L} \to G$. A *two-valued measure* on $\mathcal{L}$ is a Z-valued measure which maps $\mathcal{L}$ onto $\{0, 1\}$.

Let m be G-valued measure on a Boolean algebra $\mathcal{B}$. The kernel of m, $\operatorname{Ker} m = m^{-1}(0)$, is a weak generalized orthomodular poset (see [7] for the definition). It is an orthomodular poset (moreover: a concrete logic) iff

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$m(\mathbf{1}_{\mathcal{B}})=0$. We define a *kernel logic* as an orthomodular poset isomorphic to the kernel of some group-valued measure on a Boolean algebra. The following proposition states that introducing more measures in the definition of a kernel logic does not bring anything new.

Proposition 1.1 [11]**:** *Let $\mathcal{B}$ be a Boolean algebra and $\{m_i \mid i \in I\}$ a collection of group-valued measures on $\mathcal{B}$ satisfying $m_i(\mathbf{1}_{\mathcal{B}})=0$. Then $\bigcap_{i\in I} \operatorname{Ker} m_i$ is a kernel logic.*

We refer to [10, 11] for examples and basic properties of kernel logics. It appears that the class of kernel logics is closed under the formation of products, Boolean powers and horizontal sums. Every orthomodular poset is a homomorphic image of a kernel logic.

While the paper [11] deals mostly with real-valued measures, here we show also the use of other groups in the construction of kernel logics:

Theorem 1.2 : *Let $\mathcal{L}$ be a class of subsets (of a set X) closed with respect to the symmetric difference $A\triangle B=(A\setminus B)\cup(B\setminus A)$. Then $\mathcal{L}$ is a kernel logic.*

Proof. The set $\mathcal{L}$ with the symmetric difference forms a group. Let $\mathcal{B}$ be the field of sets generated by $\mathcal{L}$. We may consider $\mathcal{B}$ with the symmetric difference as a group, too, and $\mathcal{L}$ becomes its subgroup. We define a $(\mathcal{B}/\mathcal{L})$-valued measure m on $\mathcal{B}$ by $m(A)=\{B\in\mathcal{B} \mid A\triangle B\in\mathcal{L}\}$. Then $\mathcal{L}=\operatorname{Ker} m$. □

2 Generalized Kalmbach embedding

In [5], Kalmbach has proved that every lattice may be embedded into an orthomodular lattice. (We refer to this construction as the *Kalmbach embedding*.) The possibility of extension of this result from lattices to posets is mentioned, without proof, in [2]. Besides proving a generalization of [5], we try here to present the topic as clearly as possible. We share with J. Harding the opinion that the importance of the Kalmbach embedding has not yet been realized by a majority of experts in orthostructures and we expect further applications of this result. We present here the *generalized Kalmbach embedding* in two manners — as a collection of "continued differences" (Th. 2.2) and as a kernel logic (Th. 2.8). In order to make the method easy to understand, we exhibit a detailed example (Ex. 2.12).

Let P be a poset. As it may be embedded into a poset with unity, we shall — without any loss of generality — suppose that P has a greatest element, $\mathbf{1}_P$. In every poset, we denote by $a^\vee$ the principal filter generated by a, $a^\vee=\{b \mid b\geq a\}$. We denote by X the set of all nonvoid order filters in P, and by $\overline{X}$ the set of all principal filters in P, $\overline{X}=\{a^\vee \mid a\in P\}\subset X$. We consider X as a poset ordered by the inclusion. The collection X is closed with respect to all joins and meets (they coincide with the set-theoretical unions and intersections) and $\overline{X}$ is the set of all join irreducible elements in X. The least (resp. the greatest) element of X is $\mathbf{0}_X=\{\mathbf{1}_P\}$ (resp. $\mathbf{1}_X=P$). We define a mapping $e: P\to 2^X$ by $e(a)=\{F\in X \mid a\in F\}$. This definition is equivalent to $e(a)=a^{\vee\vee}=\{F\in X \mid a^\vee\subset F\}$.

The mapping e is an order isomorphism of P onto $e(P)=\{F^\vee \mid F\in \overline{X}\}$. The complements $X\setminus e(a)$, $a\in P$, are lattice ideals in X.

We denote by $\mathcal{K}(P)$ (resp. $\mathcal{A}(P)$) the class of subsets (resp. the field of subsets) of X generated by $e(P)$.

Remark 2.1 : Notice that if P has a least element, $\mathbf{0}_P$, then its image, $e(\mathbf{0}_P) = \{P\}$, is different from the least element of $\mathcal{K}(P)$ which is $\emptyset$.

Theorem 2.2 : *The nonempty sets from $\mathcal{K}(P)$ are exactly all the subsets of X which can be (uniquely) expressed in the form*

$$\text{(D)} \qquad e(a_1) \setminus [e(a_2) \setminus \ \ldots \ \setminus [e(a_{n-1}) \setminus e(a_n)]...],$$

where $a_1, \ldots, a_n \in P$, $a_1 > a_2 > \ldots > a_n$.

Proof. We have to prove that all the sets of the form (D) together with the empty set form a class of subsets of X.

The set X is of the form $e(\mathbf{1}_P)$.

Let $B \subsetneqq C$, where

$$B = e(b_1) \setminus [e(b_2) \setminus [\ldots]], \ b_1 > b_2 > \ldots,$$

$$C = e(c_1) \setminus [e(c_2) \setminus [\ldots]], \ c_1 > c_2 > \ldots.$$

(We exclude the trivial case $B = \emptyset$.) In order to prove that $C \setminus B$ is of the form (D), it is sufficient to verify that $b_1, b_2, \ldots, \ c_1, c_2, \ldots$ form a chain (if properly ordered). We proceed by induction. Notice that $b_1^\vee \in e(b_1) \setminus e(b_2) \subset B$. There is an even index k such that $b_1^\vee \in e(c_{k-1}) \setminus e(c_k)$, i. e. $c_{k-1}^\vee \leq b_1^\vee$, $c_k^\vee \not\leq b_1^\vee$. We obtain the orderings $b_2 < b_1 \leq c_{k-1} < \ldots < c_1$. It remains to order the elements $b_2, b_3, \ldots, \ c_k, c_{k+1}, \ldots$ into a chain (with b_2 as its greatest element). We have

$$B = e(b_1) \setminus [e(b_2) \setminus [e(b_3) \setminus \ldots]] = e(b_1) \cap B \subseteq$$

$$e(b_1) \cap C = e(b_1) \setminus [e(c_k) \setminus [e(c_{k+1}) \setminus \ldots]],$$

which implies

$$e(c_k) \setminus [e(c_{k+1}) \setminus \ldots] \subseteq e(b_2) \setminus [e(b_3) \setminus \ldots].$$

The latter inclusion is of the same form as the original inclusion $B \subset C$, but with a smaller number of entries. Repeating the same procedure, we see that all the elements $b_1, b_2, \ldots, \ c_1, c_2, \ldots$ form a chain and $C \setminus B$ is of the form (D). $\square$

Proposition 2.3 : *The mapping $e : P \to \mathcal{K}(P)$ preserves all joins and meets existing in P.*

We call the above construction which embeds a poset P into the orthomodular poset $\mathcal{K}(P)$ the *generalized Kalmbach embedding.*

Remark 2.4 : Joins and meets in $\mathcal{K}(P)$ do not coincide with the set-theoretical unions and intersections.

Proposition 2.5 : *If P is a lattice, $\mathcal{K}(P)$ is also a lattice, and our construction coincides with the Kalmbach embedding of $P \cup \{\mathbf{0}\}$, where $\mathbf{0}$ is the least element of $P \cup \{\mathbf{0}\}$.*

Corollary 2.6 : *The result of the Kalmbach embedding is a concrete logic.*

Remark 2.7 : The latter corollary is mentioned and applied by Harding in [2]. However, there is a small mistake in his proof. He takes only $\overline{X}$ for the domain of the set representation of $\mathcal{K}(P)$. To see that this is not sufficient, one may take $P \cong 2^2 \setminus \{\mathbf{0}\}$, i. e. $P = \{a, b, \mathbf{1}\}$ with no other orderings than $a < \mathbf{1}, b < \mathbf{1}$. Then $\mathcal{K}(P)$ is isomorphic to the orthomodular lattice MO_2 (see [6]) and the set representation of $\mathcal{K}(P)$ requires at least 4 two-valued measures, while $\operatorname{card} \overline{X} = \operatorname{card} P = 3$. Thus we provide here, as far as we know, the first correct proof of the concreteness of $\mathcal{K}(P)$ even for the case when P is a lattice. Let us mention that $\mathcal{K}(P)$ may be represented also by a class of subsets of the set $\{a^\vee \cup b^\vee \mid a, b \in P\} \subset X$.

Now we shall show that the embedding described above results in a kernel logic. For each $F \in X$, we denote by s_F the two-valued measure $s_F : \mathcal{A}(P) \to \{0, 1\} \subset Z$ concentrated in F, i. e.

$$s_F(A) = 1 \iff F \in A.$$

For each $F, G \in X$ we define a measure $m_{F,G} : \mathcal{A}(P) \to Z$ by

$$m_{F,G} = s_F + s_G - s_{F \cup G} - s_{F \cap G}.$$

(If F, G are comparable, then $m_{F,G} = 0$; it is not necessary to exclude this case.)

Theorem 2.8 : *$\mathcal{K}(P)$ is a kernel logic, $\mathcal{K}(P) = \bigcap_{F,G \in X} \operatorname{Ker} m_{F,G}$.*

Proof. As all $e(a)$, $a \in P$, are principal filters corresponding to join irreducible elements, the measures $m_{F,G}$ vanish on them. Thus $\operatorname{Ker} m_{F,G}$ contains $e(P)$ and also the class of subsets $\mathcal{K}(P)$ generated by it.

Suppose that $A_1 \in \bigcap_{F,G \in X} \operatorname{Ker} m_{F,G} \setminus \{\emptyset\}$. As an element of $\mathcal{A}(P)$, A_1 can be expressed as a polynomial with a (finite) set $E(A_1)$ of entries of the form $e(a) = a^{\vee\vee}$, $a \in P$. In the standard form, A_1 is expressed as a finite (disjoint) union of finite intersections of the form

$$\bigcap_i b_i^{\vee\vee} \cap \bigcap_j (X \setminus c_j^{\vee\vee}) = (\bigcup_i b_i^\vee)^\vee \cap \bigcap_j (X \setminus c_j^{\vee\vee}),$$

where $b_i, c_j \in P$. If this set is nonempty, $\bigcup_i b_i^\vee$ is its least element. (We take $\bigcup_{i \in \emptyset} b_i^\vee = \mathbf{0}_X = \{\mathbf{1}_P\}$ if this situation occurs.) Therefore there is a finite set $I \subseteq A_1$ such that $A_1 \subseteq \bigcup_{F \in I} F^\vee$. The set I can be chosen such that it contains only minimal elements of A_1. We assume this case. For each $G, H \in I$, $m_{G,H}(A_1) = 0$ implies $G \cap H \in A_1$. Thus I contains a single element, F_1, which is the least element of A_1. If F_1 is a nontrivial union of $G, H \in X$, i. e. if $F_1 = G \cup H$, $\emptyset \subsetneq G \subsetneq F_1$, $\emptyset \subsetneq H \subsetneq F_1$, then $m_{G,H}(A_1) = -1$, a contradiction. This implies that $F_1 \in \overline{X}$ and there is an $a_1 \in P$ such that $F_1 = a_1^\vee$, $F_1^\vee = e(a_1)$. As $A_1 \subseteq e(a_1)$, we may write $A_1 = e(a_1) \setminus A_2$, where $A_2 = e(a_1) \setminus A_1 \in \bigcap_{F,G \in X} \operatorname{Ker} m_{F,G}$. We have $A_2 \in \mathcal{K}(P)$ iff $A_1 \in \mathcal{K}(P)$. We proceed by induction, $A_2 = e(a_2) \setminus A_3$, ... As all $e(a_i)$, $i = 1, 2, \ldots$, form a decreasing sequence and they belong to the (finite) Boolean subalgebra of $\mathcal{A}(P)$ generated by the set $E(A_1)$ of entries of the polynomial expressing A_1, we obtain $A_n = \emptyset$ for some finite n. Consequently, $\mathcal{K}(P)$ contains $A_n, A_{n-1}, \ldots, A_1$. $\square$

Remark 2.9 : As we did not use Th. 2.2, we may consider the proof of Th. 2.8 as an alternative description of the generalized Kalmbach embedding.

In [3], Janowitz introduced the class of *constructible lattices* — it is the smallest class of lattices containing all Boolean algebras and closed under products and horizontal sums. As all constructible lattices can be obtained by the Kalmbach embedding, we have an alternative proof of the following corollary, already stated in [11].

Corollary 2.10 : *All constructible lattices are kernel logics.*

The logics obtained by the generalized Kalmbach embedding possess also the following interesting property. (A measure m is called *faithful* if $m(a) = 0$ implies $a = \mathbf{0}$.)

Theorem 2.11 : *Let P be a poset and G a group. Each mapping $f : P \setminus \{\mathbf{0}_P\} \to G$ extends uniquely to a G-valued measure $m : \mathcal{K}(P) \to G$ such that $m \mid P = f$. In particular, if $G = R$ and f is increasing (resp. strictly increasing), then m is non-negative (resp. non-negative and faithful).*

Proof. It is a direct consequence of Th. 2.2. $\square$

Example 2.12 : Let us demonstrate the generalized Kalmbach embedding on the least poset P with unity which is not a lattice — see Fig. 1. Principal filters (=elements of $\overline{X}$) are marked by black circles in the lattice X of filters of P. The result $\mathcal{K}(P)$ of the generalized Kalmbach embedding is the least orthomodular poset which is not a lattice. In its Haase diagram, elements $e(a), e(a)'$ are doubled to avoid intricate crossings. The Greechie diagram of $\mathcal{K}(P)$ is supplied for those who are acquainted with this technique. We show a representation of $\mathcal{K}(P)$ as the kernel of a Z^2-valued measure m on 2^X. (Among the measures used in the proof of Th. 2.8, only two distinct measures are nonzero.)

3 Logics satisfying the outer point condition

In this section we prove that the outer point condition is sufficient for an orthomodular poset to be a kernel logic. This allows to represent every group as the automorphism group of a kernel logic.

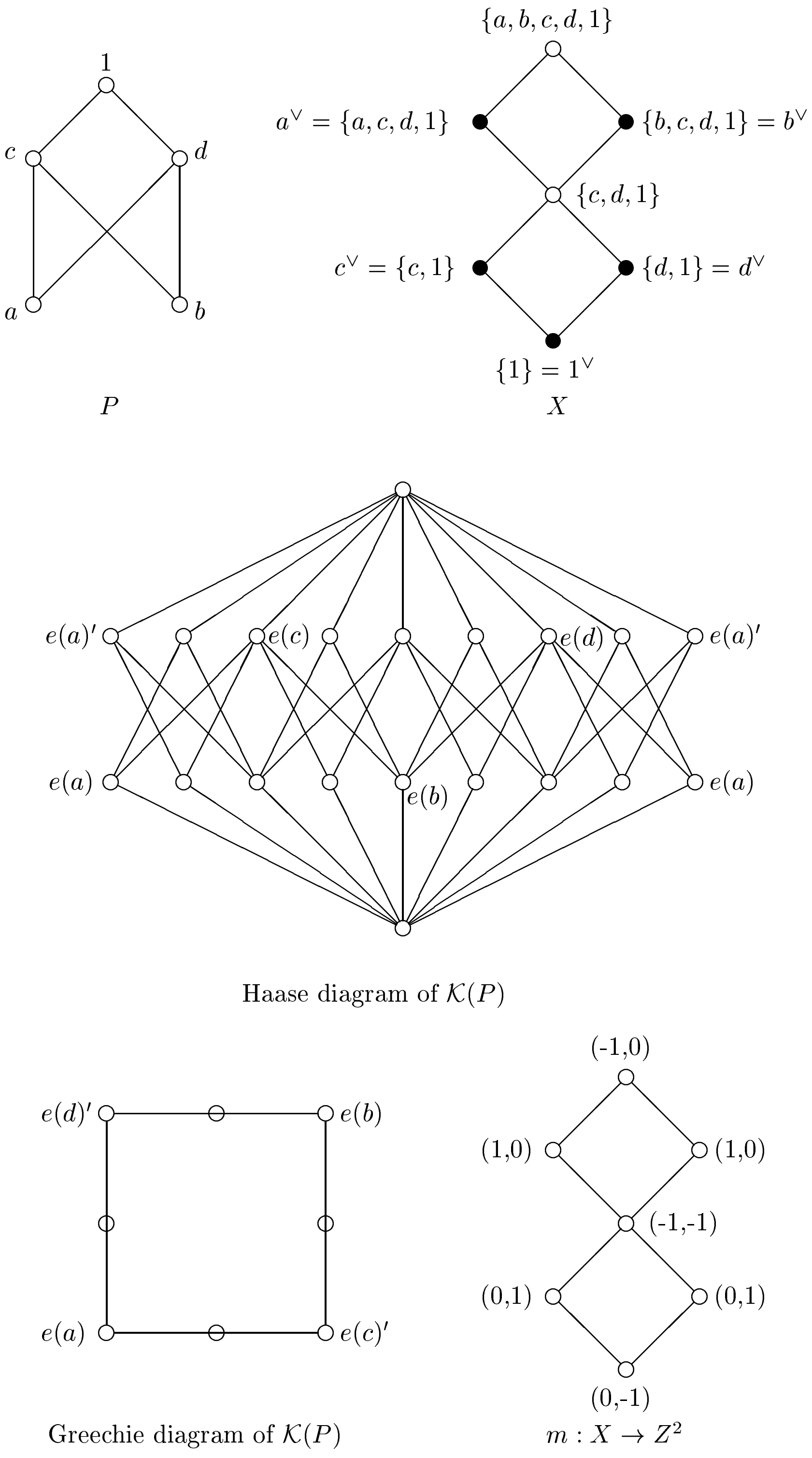
Throughout this section, $\mathcal{L}$ denotes on orthomodular poset, We say that $a \in \mathcal{L}$ is an *atom* if $\{b \in \mathcal{L} \mid \mathbf{0}_{\mathcal{L}} < b < a\}$ is empty. A *block* is a maximal Boolean subalgebra of $\mathcal{L}$. Every orthomodular poset is a union of its blocks. From now on, $\mathcal{B}\ell$ denotes the family of all blocks of $\mathcal{L}$ and $\mathcal{A}t$ is the set of all atoms of $\mathcal{L}$.

An *outer point* is an atom which belongs to exactly one block. If each block of $\mathcal{L}$ contains an outer point, we say that $\mathcal{L}$ satisfies the *outer point condition*. We say that $\mathcal{L}$ is *chain-finite* if all chains in $\mathcal{L}$ are finite.

Theorem 3.1 : *A chain-finite orthomodular poset satisfying the outer point condition is a kernel logic.*

Proof. We choose in each $A \in \mathcal{B}\ell$ an outer point p_A. Denote $P = \{p_A \mid A \in \mathcal{B}\ell\}$. We first have to find an (appropriate) set representation of $\mathcal{L}$. We define a collection of two-valued measures (on $\mathcal{L}$) $X = \{s_0\} \cup \{s_a \mid a \in \mathcal{A}t \setminus P\} \cup \{s_{a,b} \mid a, b \in \mathcal{A}t \setminus P, a \not\perp b\}$, where

0. $s_0^{-1}(1) = \bigcup_{A \in \mathcal{B}\ell} p_A^{\vee}$,



1
c
d
a
b
P
{a, b, c, d, 1}
a^∨ = {a, c, d, 1}
{b, c, d, 1} = b^∨
{c, d, 1}
c^∨ = {c, 1}
{d, 1} = d^∨
{1} = 1^∨
X
e(a)′
e(c)
e(d)
e(a)′
e(a)
e(b)
e(a)
Haase diagram of 𝒦(P)
e(d)′
e(b)
e(a)
e(c)′
Greechie diagram of 𝒦(P)
(-1,0)
(1,0)
(1,0)
(-1,-1)
(0,1)
(0,1)
(0,-1)
m : X → Z^2

Fig. 1

1. $s_a^{-1}(1) = a^\vee \cup \bigcup_{A \in \mathcal{B}\ell, a \notin A} \ p_A^\vee$,
2. $s_{a,b}^{-1}(1) = a^\vee \cup b^\vee \cup \bigcup_{A \in \mathcal{B}\ell, \ a,b, \notin A} \ p_A^\vee$.

The mapping $r : d \mapsto \{s \in X \mid s(d) = 1\}$ is an isomorphism of $\mathcal{L}$ onto the class $r(\mathcal{L})$ of subsets of X. (To prove that $r(\mathcal{L})$ is a class of subsets, it suffices to find, for each $a, b \in \mathcal{L}$, $a \not\perp b$, a two-valued measure s such that $s(a) = s(b) = 1$ (see [14]). It is sufficient to do this in the case of $a, b \in \mathcal{A}t$. If $a, b \in P$, we take s_0; if $a \notin P$, $b \in P$, we take s_a; if $a, b \notin P$, we take $s_{a,b}$.)

For each $s \in X$, let $\overline{s}$ denote the two-valued measure on 2^X concentrated in s. Notice that, for $c \in \mathcal{L}$ and $s \in X$, we have $\overline{s}(r(c)) = s(c)$. For each $a, b \in \mathcal{A}t \setminus P$, $a \not\perp b$, we define a Z-valued measure $m_{a,b}$ on 2^X by

$$m_{a,b} = \overline{s_a} + \overline{s_b} - \overline{s_{a,b}} - \overline{s_0}.$$

We claim that $r(\mathcal{L}) = \bigcap_{a,b \in \mathcal{A}t \setminus P, a \not\perp b} \operatorname{Ker} m_{a,b}$. The verification that $r(\mathcal{L})$ is a subset of all $\operatorname{Ker} m_{a,b}$ is easy. Let $S \in \bigcap_{a,b \in \mathcal{A}t \setminus P, a \not\perp b} \operatorname{Ker} m_{a,b}$. Notice that $S \in r(\mathcal{L})$ iff $X \setminus S \in r(\mathcal{L})$. Taking the complement if necessary, we may suppose that $s_0 \notin S$.

For each $a, b \in \mathcal{A}t \setminus P$, $a \not\perp b$, the equality $m_{a,b}(S) = 0$ implies that the intersection $S \cap \{\overline{s_a}, \overline{s_b}, \overline{s_{a,b}}\}$ is of one of the three forms $\emptyset$, $\{\overline{s_a}, \overline{s_{a,b}}\}$, $\{\overline{s_b}, \overline{s_{a,b}}\}$. Thus S can be expressed as a union of sets of the form

$$\{\overline{s_a}\} \cup \{\overline{s_{a,b}} \mid b \in \mathcal{A}t \setminus P, b \not\perp a\} = r(a),$$

i. e.

$$S = \bigcup_{a \in M} r(a)$$

for some set $M \subset \mathcal{A}t \setminus P$ which contains no nonorthogonal pair. As the elements of M are mutually orthogonal, they are contained in a single block of $\mathcal{L}$ (see [12]). Further, the sets $r(a)$, $a \in M$, are mutually disjoint. The element $e = \bigvee_{a \in M} a \in \mathcal{L}$ satisfies $S = r(e) \in r(\mathcal{L})$. $\square$

Orthomodular lattices satisfying the outer point condition were used in [4] for the construction of an orthomodular lattice with given automorphism group. Thus we have the following corollary.

Corollary 3.2 : *Every group is isomorphic to the automorphism group of some kernel logic.*

This result is a bit surprising if compared with the situation in Boolean algebras (see [8]): While only some special groups may be represented as automorphism groups of Boolean algebras, every group may be represented as the automorphism group of a kernel of a measure on a Boolean algebra.

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