Constrained fuzzy arithmetic

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In the standard fuzzy arithmetic, we may observe a phenomenon known already from interval arithmetic: The vagueness of quantities always increases. E.g., for a variable X, the difference $X - X$ need not be zero and $10X - 9X$ is much more vague than X. Klir [1, 2] suggests an alternative—constrained fuzzy arithmetic—which reduces this effect using our knowledge that X represents the same quantity throughout the expression. The difference may be shown at the following example:

Example 1 *To rent a bike, one has to pay a deposit between \$200 and \$250. If we rented 10 bikes and returned 9* **other** *bikes (rented earlier), we have paid the difference of the deposits which might be any number from the interval*

$$10 \cdot [200, 250] - 9 \cdot [200, 250] = [2000, 2500] - [1800, 2250] = [-250, 700] .$$

(The negative result means that we could even receive money instead of payment!) On the other hand, if we rented 10 bikes and returned 9 of **these**, *then apparently we had to pay an amount from the original interval* $[200, 250]$.

Standard interval and fuzzy arithmetic do not distinguish these cases and give always the first result presented in the example. While the standard com-

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putation with an interval X gives

$$X + X = \{x + y : x \in X, y \in X\} ,$$

the alternative approach uses the formula

$$(X + X)_{\mathrm{CFA}} = \{x + x : x \in X\} .$$

(Similarly for other operations.) Recall that there is no difference if the two arguments are *different* quantities, say X, Y:

$$X + Y = (X + Y)_{\mathrm{CFA}} = \{x + y : x \in X, y \in Y\} .$$

Different results are obtained only in case when some of the variables appears repeatedly. Fuzzy intervals can be represented by their α-cuts which are closed intervals, thus the above rules can be canonically extended to them. Such arithmetic, called *constrained fuzzy arithmetic*, possesses all laws of arithmetic of reals. This seems to be desirable in many applications.

Little attention was paid to the problems of implementation of constrained fuzzy arithmetic, especially to its computational efficiency. In [3, 4], we point out the related problems and outline the ways of their solution. The main problem is that a complex expression cannot be decomposed to a sequence of binary operations in constrained fuzzy arithmetic. E.g., consider the following formula with intervals X, Y, Z:

$$(X + (Y \cdot (Z + X)))_{\mathrm{CFA}} = \{x + (y \cdot (z + x)) : x \in X, y \in Y, z \in Z\} .$$

The sum $Z + X$ cannot be computed separetely from the whole expression. Finding the bounds of the resulting intervals requires to find extremes of the expression $x + (y \cdot (z + x))$. Here it is not difficult, but one may easily arrive at problems that are not analytically solvable and also numerical approximations may be difficult. To reduce the computational complexity, it is desirable to look for a simplification of the formula. E.g.,

$$((X + Y) \cdot (X - Y))_{\mathrm{CFA}} = ((X \cdot X) - (Y \cdot Y))_{\mathrm{CFA}} = (X \cdot X)_{\mathrm{CFA}} - (Y \cdot Y)_{\mathrm{CFA}} .$$

The first equality is obtained from arithmetic of reals; then we split the expression to subexpression which have disjoint sets of variables. In this case, we may use the standard fuzzy arithmetic at least partially, thus reducing the number of variables over which we search for extremes in the constrained fuzzy arithmetic.

The problem of complexity described above was already encountered in interval arithmetic. What is new in constrained fuzzy arithmetic is the possibility to optimize the calculations for fuzzy intervals [5]: Repeating the interval calculation for a sequence of α-cuts, we do not need to use a blind search of extremes for each value of α, but we may use the results from the higher cut and simplify the procedure [4].

References

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