Homework from Fuzzy Logic. Choose the basic or advanced level.

Basic level
Exercises from conversions of representations of fuzzy sets and generators of fuzzy negations.

Exercise 2.1 Fuzzy set $B$ is given by its collection of cuts:

\[
\mathcal{R}_B(\alpha) = \begin{cases} 
\{A, B, C, D, E, F\}, & \alpha = 0, \\
\{A, B, C, D, E\}, & \alpha \in (0, 0.5), \\
\{A, B, C\}, & \alpha \in [0.5, 0.6), \\
\{A, B, C, D\}, & \alpha \in [0.6, 0.7), \\
\{A, B\}, & \alpha \in [0.7, 0.9), \\
\{A\}, & \alpha \in [0.9, 1].
\end{cases}
\]

Find its vertical representation.

Exercise 2.2 Fuzzy set $C$ is given by its collection of cuts:

\[
\mathcal{R}_C(\alpha) = \begin{cases} 
\mathbb{R}, & \alpha = 0, \\
(\alpha, 6 - 2\alpha^2), & \alpha \in (0, 1].
\end{cases}
\]

Find its vertical representation.

Exercise 2.3 Fuzzy set $D$ is given by its collection of cuts:

\[
\mathcal{R}_D(\alpha) = \begin{cases} 
\mathbb{R}, & \alpha = 0, \\
[-2 + \alpha, 2 - \alpha], & \alpha \in (0, 1), \\
\{0\}, & \alpha = 1.
\end{cases}
\]

Find its vertical representation.

Exercise 2.4 Find for which $\lambda$ the formula

\[
\gamma_\lambda^{\alpha} = \frac{1 - \alpha}{1 + \lambda \alpha}
\]

defines a fuzzy negation $\gamma$. Draw a typical graph.

Exercise 2.5 Find the fuzzy negation with generator $i(\alpha) = \alpha^2$. Draw its graph.

Advanced level
Exercises from generators of negations.

Exercise 2.6 Decide whether the following statement is true:
For every two fuzzy negations $\gamma_1, \gamma_2$, there is an increasing bijection $i: [0, 1] \to [0, 1]$ such that $\gamma_2^{\alpha} = i^{-1}(\gamma_1^{i(\alpha)})$ for all $\alpha \in [0, 1]$.

Exercise 2.7 Try to find a generator of the standard fuzzy negation, different from the identity.