Homework from Fuzzy Logic. Choose the basic or advanced level.

Basic level
Exercises from conversions of representations of continuous fuzzy sets.

Exercise 2.1 Fuzzy set $A$ has the following vertical representation:

$$
\mu_A(x) = \begin{cases} 
1, & x \in [9, 11], \\
\frac{x - 6}{3}, & x \in [6, 9], \\
12 - x, & x \in [11, 12], \\
0, & \text{otherwise}.
\end{cases}
$$

Find the horizontal representation of this fuzzy set.

Exercise 2.2 Fuzzy set $B$ is given by its collection of cuts:

$$
R_B(\alpha) = \begin{cases} 
\mathbb{R}, & \alpha = 0, \\
[2 + 3\alpha, 7 - 2\alpha], & \alpha \in (0, 1).
\end{cases}
$$

Find its vertical representation.

Exercise 2.3 Fuzzy set $C$ is given by its collection of cuts:

$$
R_C(\alpha) = \begin{cases} 
\mathbb{R}, & \alpha = 0, \\
[2 + \alpha, 6 - 2\alpha], & \alpha \in (0, 1).
\end{cases}
$$

Find its vertical representation.

Exercise 2.4 Fuzzy set $D$ is given by its collection of cuts:

$$
R_D(\alpha) = \begin{cases} 
\mathbb{R}, & \alpha = 0, \\
[2 + \alpha, 6 - 2\alpha], & \alpha \in (0, 1), \\
\{4\}, & \alpha = 1.
\end{cases}
$$

Find its vertical representation.

Advanced level
Exercises from generators of negations.

Exercise 2.5 Prove that any (strong) fuzzy negation can play the role of the standard negation in the representation theorem for fuzzy negations.

Exercise 2.6 Try to find a generator of the standard fuzzy negation, different from the identity.