Independence in D-posets.

Eva Drobná, Ferdinand Chovanec[1], Olga Nánásiová[2]

1 Department of Natural Sciences, Armed Forces Academy
Liptovský Mikuláš, Slovak Republic
e-mail: drobna@aos.sk, chovanec@aos.sk

2 Department of Mathematics and Descriptive Geometry
Faculty of Civil Engineering
Slovak Technical University, Bratislava, Slovak Republic
e-mail: olga@vox.svf.stuba.sk

Problems and results concerning the definition of the conditional probability, conditional system and independence of events on orthomodular lattices are extended to a more general structural basis, on a D-poset. Substantial differences in a comprehensive characterization of a conditional system are given.

Conditional probabilities on a classical measurable space are studied in several different ways, but result in equivalent theories. The classical probability theory does not decsribe the causality model.

The situation changes when non-standard spaces are considered. For example, it is well known that the set of random events in quantum mechanic experiments is a more general structure than Boolean algebra. In the quantum logic approach the set of random events is assumed to be a quantum logic L. Such a model can be found not only in the quantum theory, but also in economics, biology, etc.

A conditional state on a D-poset [1] (as it is the most general algebraic structure describing random events) is defined using Renyi's approach (or Bayesian principle). This approach helps us to define such independence of events that addmits the situation completely different from that one known

in the classical probability theory. Namely, if an event A is independent of an event B, then the event B can be dependent on the event A [2], [3] (problem of causality).

Comparison of conditional probability on regular and non-regular quantum structures is also specified. As it will be shown itself, the question of a conditional set, which is essential for the definition of a conditional state as a function of two variables, is much more complicated than in orthomodular lattices where the maximal conditional set contains all non-zero elements. The characterization of a maximal conditional set in a D-poset cannot be such plain.

Acknowledgments. The authors were supported by the Slovak Academy of Sciences via the project Center of Excellence - Physics of Information, by the grant I/2/2005 and grants APVV-0071-06, APVV-0375-06, and VEGA 1/4024/07.

References

[1] Chovanec, F. and Kôpka, F.: D-posets. In: *Handbook of Quantum Logic and Quantum Structures: Quantum Structures*. Eds. Engesser, K., Gabbay, D.M., and Lehmanm, D., Elsevier (2007), 367–428.

[2] Nánásiová O.: *A note on the independent events on aquantum logic*, Busefal 76 (1998), 53–57.

[3] Nánásiová O.: *Principle conditioning*. Int. Jour. of Theor. Phys. 43, No. 7-8 (2004), 1757–1767.