Non-orthomodular lattice effect algebras possessing state

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Common generalizations of MV-algebras [2] and orthomodular lattices are lattice effect algebras [4]. An *effect algebra* $(E; \oplus, 0, 1)$ is a set E with two special elements $0, 1$ and a partial binary operation $\oplus$ which is commutative and associatiove at which these equalities hold if one of their sides exists. Moreover, to every element $a \in E$ there exists a unique element $a' \in E$ with $a \oplus a' = 1$ and if $a \oplus 1$ exists then $a = 0$. In every effect algebra we can define a partial order by $a \leq b$ iff there exists $c \in E$ with $a \oplus c = b$ (we set $c = b \ominus a$). If $(E; \leq)$ is a lattice (a complete lattice) then $(E; \oplus, 0, 1)$ is called a *lattice effect algebra* (*complete lattice effect algebra*).

Effect algebras are a generalization of many structures which arise in quantum physics (see [1]) and in mathematical economics (see [3]). In approach to the mathematical foundations of physics the fundamental notions are states, observables and symmetries. D.J. Foulis in [5] showed how effect algebras arise in physics and how they can be used to tie together the observables, states and symmetries employed in the study of physical systems.

In spite of the fact that effect algebras are very natural algebraic structures to be carriers of states and probability measures, in above mentioned non-classical cases of sets of events, there are even finite effect algebras admitting no states, hence no probabilities. The smallest of them has only nine elements (see [8]). Some possibility to eliminate such unfavourable situation have modular complete lattice effect algebras (see [9]).

We can prove

Theorem 1 [7] Let E be an Archimedean atomic lattice effect algebra, $c \in C(E)$, c finite in E, $c \neq 0$, $[0, c]$ a modular lattice. Then there exists an (o)-continuous state ω on E, which is subadditive.

Moreover, for non-orthomodular (i.e., $S(E) \neq E$) lattice effect algebras E we obtain

Theorem 2 [6] Let E be a sharply dominating Archimedean atomic lattice effect algebra with $B(E) \neq C(E)$. Then there exists an extremal (o)-continuous state ω on E, which is subadditive.

Theorem 3 [7] Let E be a modular Archimedean atomic lattice effect algebra with $S(E) \neq E$. Then there exists an (o)-continuous state ω on E, which is subadditive.

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