Globally Convergent Range Image Registration by Graph Kernel Algorithm

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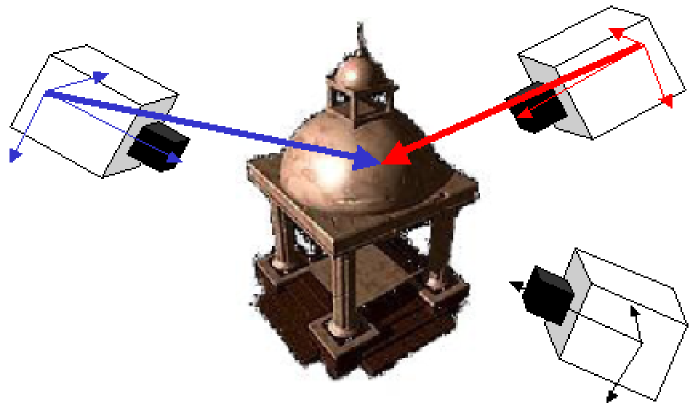
Range Image Registration Problem



Difficulties

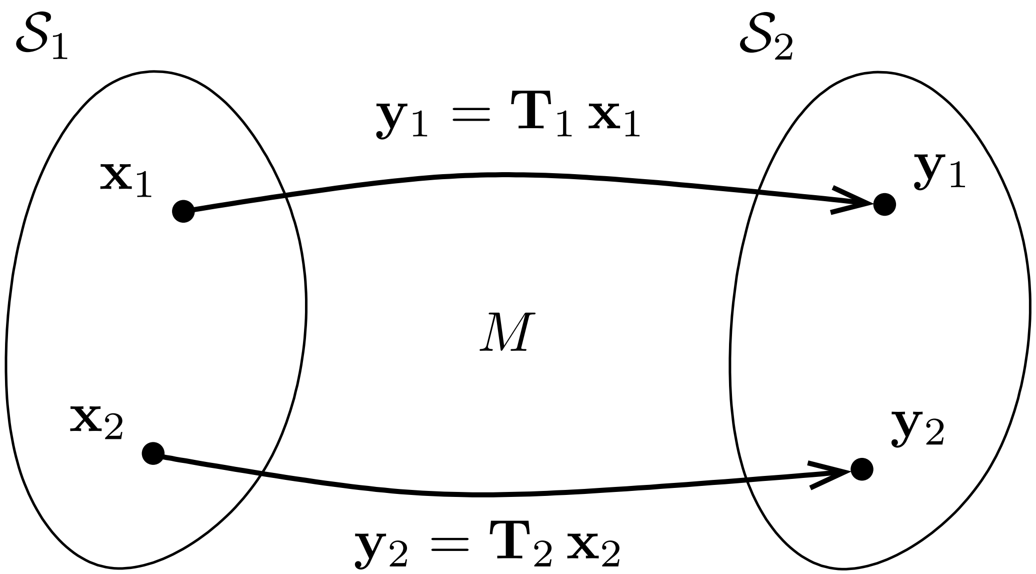
- Half-occlusion ⇒ solutions are 'partial' matchings
- Finite resolution ⇒
  1. 'true correspondences' are not discrete
  2. surface discretization is not viewpoint-invariant
- Occluding boundary artefacts ⇒ robust methods

In This Talk

- A **robust** matching method
- for **partial (incomplete) matchings**,
- which is algorithmically **efficient**.
- This is possible in discrete optimization framework of **graph kernels**.

Assumptions: rigid objects, no texture information

Posing the Surface Registration Problem



putative correspondence:
$(\mathbf{x}_i,\ \mathbf{y}_i;\ \mathbf{T}_i)$

Task: Find matching $M : \mathcal{S}_1 \to \mathcal{S}_2$ and registration parameters $\mathbf{T} = \{\mathbf{R},\ \mathbf{t}\}$ under:

- **similarity of invariants** F:

$$(\mathbf{x}, \mathbf{y}) \in M \quad \text{if} \quad F(\mathbf{x}) \sim F(\mathbf{y}) \qquad \mathbf{x} \in \mathcal{S}_1,\ \mathbf{y} \in \mathcal{S}_2$$

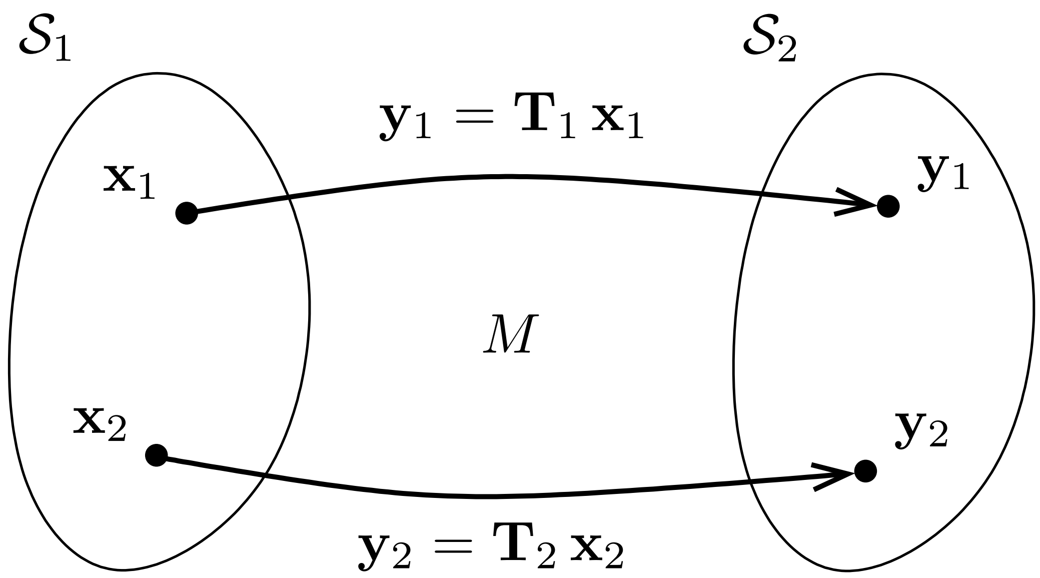
- **geometric compatibility of covariants** $\mathbf{x}_i$, $\mathbf{n}_i$, etc:

$$\mathbf{T}_p = \mathbf{T}_q\ (= \mathbf{T}) \quad \text{for all} \quad p,\ q \in M$$

- **uniqueness constraint:** each point $\mathbf{x}_j$ is matched at most once

Checking Compatibility of Covariants is Cheap

- checking $\mathbf{T}_1 = \mathbf{T}_2 = \mathbf{T}$ does need the knowledge of $\mathbf{T}_1$, $\mathbf{T}_2$



A single correspondence does not provide all parameters of $\mathbf{T}$, but a pair overconstrains it!

- given positions $\mathbf{x}_i$, $\mathbf{y}_i$ and normal vectors $\mathbf{n}_i$, $\mathbf{m}_i$, we know

$$\mathbf{y}_i = \mathbf{R}_i\,(\mathbf{x}_i - \mathbf{t}), \quad \mathbf{m}_i = \mathbf{R}_i\,\mathbf{n}_i, \quad i = 1, 2$$

$\mathbf{T}_1 = \mathbf{T}_2$ iff there is a special orthogonal matrix $\mathbf{R}$ such that

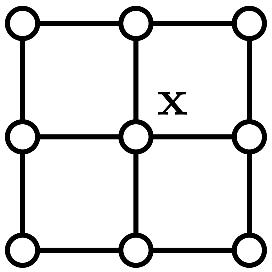
$$[\mathbf{y}_2 - \mathbf{y}_1,\, \mathbf{m}_2,\, \mathbf{m}_1] = \mathbf{R}\,[\mathbf{x}_2 - \mathbf{x}_1,\, \mathbf{n}_2,\, \mathbf{n}_1]$$

$\Rightarrow$ a Yes/No $\mathbf{T}_1 = \mathbf{T}_2$ compatibility condition over correspondence pairs

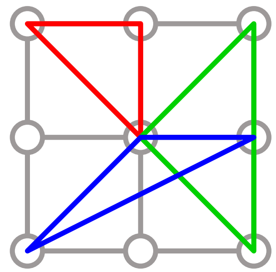
- $\mathbf{R}$: 3 parameters $\Rightarrow$ highly redundant condition
- a weaker necessary condition is, e.g. $\|\mathbf{y}_2 - \mathbf{y}_1\| = \|\mathbf{x}_2 - \mathbf{x}_1\|$
- together with $\mathbf{n}$ we also use a splash-like structure matrix (see the paper)

Invariant Features & Their Similarity

elementary oriented triangles

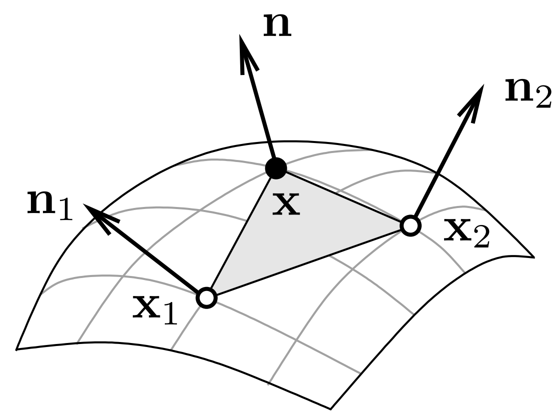


3×3 image neighborhood



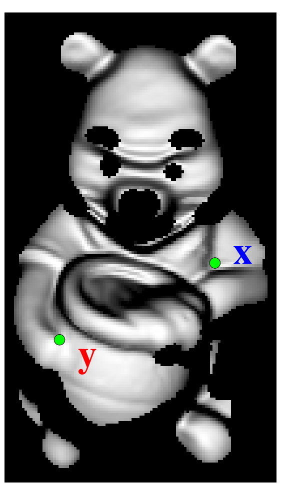
3 of all 24 triangles

for each triangle i: triple feature

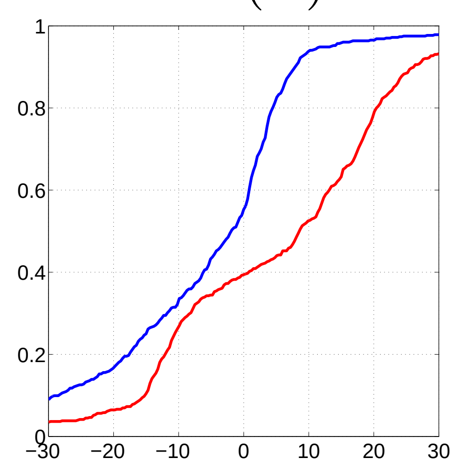


$$f_i(\mathbf{x}) = \frac{\det[\mathbf{n},\, \mathbf{n}_1,\, \mathbf{n}_2]}{\|(\mathbf{x}_1 - \mathbf{x}) \times (\mathbf{x}_2 - \mathbf{x})\|}$$

Point neighborhood gives a large collection $F(\mathbf{x}) = \{f_i;\ i = 1, \ldots, t\}$



$\mathrm{CDF}(F)$



$$\mathrm{simil}\,(F(\mathbf{x}),\, F(\mathbf{y})) = \mathrm{KS}\big(\mathrm{CDF}(F(\mathbf{x})),\, \mathrm{CDF}(F(\mathbf{y}))\big)$$

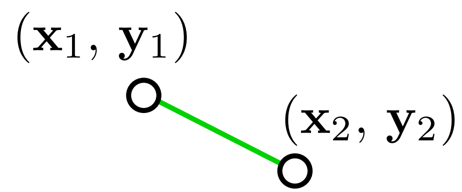
similarity $\sim$ Kolmogorov-Smirnov distance between feature distributions

in fact sensitivity interval $[\mathrm{KS} - \delta(\mathrm{KS}),\, \mathrm{KS}]$

Representing the Matching Problem

Geometric Compatibility Graph $\mathcal{G}_C$:

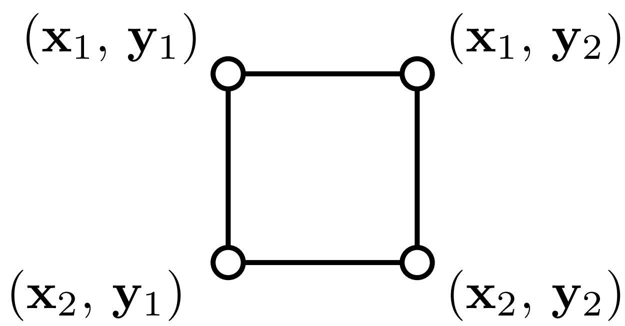
putative correspondences $(\mathbf{x}_1,\, \mathbf{y}_1)$ and $(\mathbf{x}_2,\, \mathbf{y}_2)$ are incompatible if $\mathbf{T}_1 \neq \mathbf{T}_2$



green for ‘geometric’

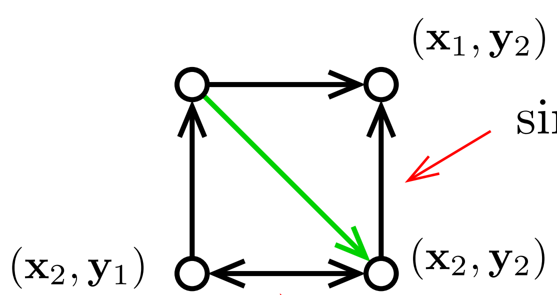
Uniqueness Graph $\mathcal{G}_U$:

Choose either $(\mathbf{x}_1,\, \mathbf{y}_1)$ or $(\mathbf{x}_1,\, \mathbf{y}_2)$ but never both



This is the line graph of a complete bipartite graph

Given data: The union of $\mathcal{G}_C \cup \mathcal{G}_U$ is oriented by similarity of invariant features F:



$$\mathrm{simil}\big(F(\mathbf{x}_1), F(\mathbf{y}_2)\big) \gg \mathrm{simil}\big(F(\mathbf{x}_2), F(\mathbf{y}_2)\big)$$

q is strongly better (intervals do not overlap)

$$\mathrm{simil}\big(F(\mathbf{x}_2), F(\mathbf{y}_1)\big) \sim \mathrm{simil}\big(F(\mathbf{x}_2), F(\mathbf{y}_2)\big)$$

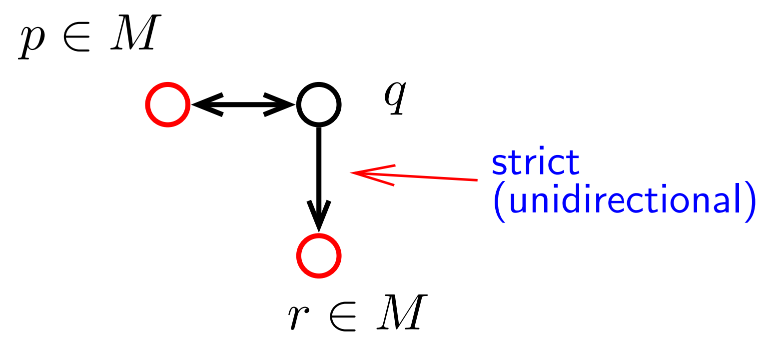
we do not know which is better (overlapping intervals)

Strict Sub-Kernel of an Oriented Graph

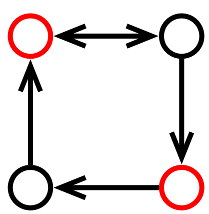
Goal: To define which is 'the best solution.'

Def. [Strict Sub-Kernel, SSK]

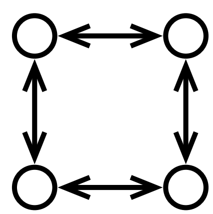
An independent vertex subset M is a strict sub-kernel of oriented graph $\mathcal{G}$ if every successor of every $p \in M$ has a strict successor in M.



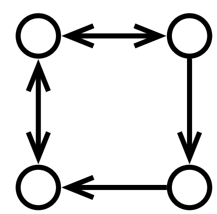
Examples:



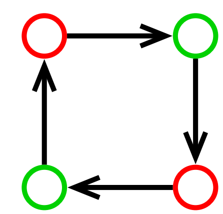
has one SSK



has no SSK



has no SSK



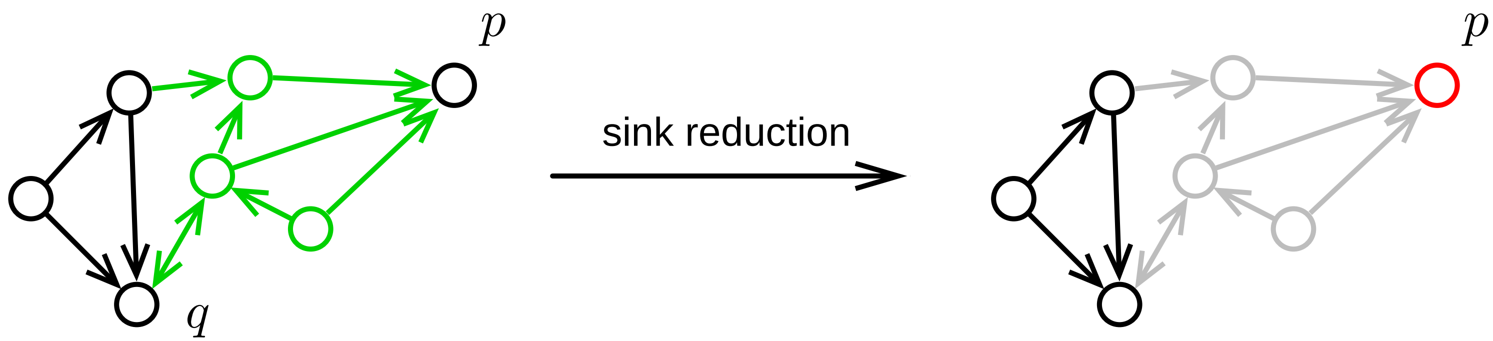
has 2 SSKs

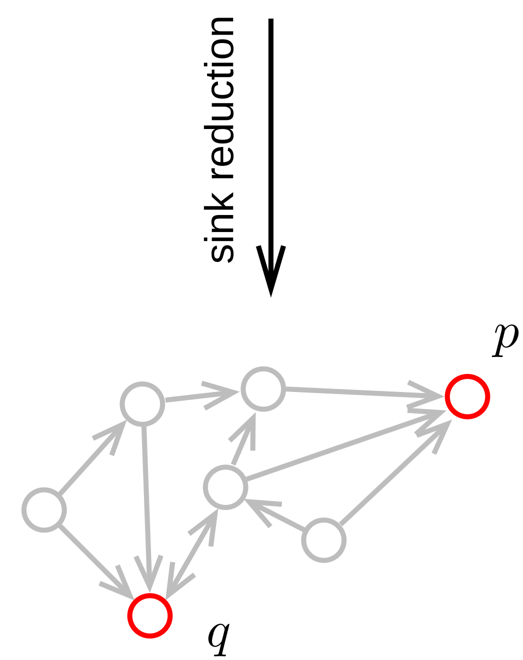
T: If every even circuit of $\mathcal{G}$ has a bidirectional arc $\Rightarrow$ **there is at most one** maximal SSK.

- condition guaranteed for orientations induced by interval overlap
- SSK can be incomplete if data insufficient or contradicting the model
  explains part of data that is consistent with prior model (geometric consistency, uniqueness)
- SSK is robust to small data perturbations

Strict Sub-Kernel Algorithm

Sink reduction algorithm: Successive simplifying transformations to equivalent problems



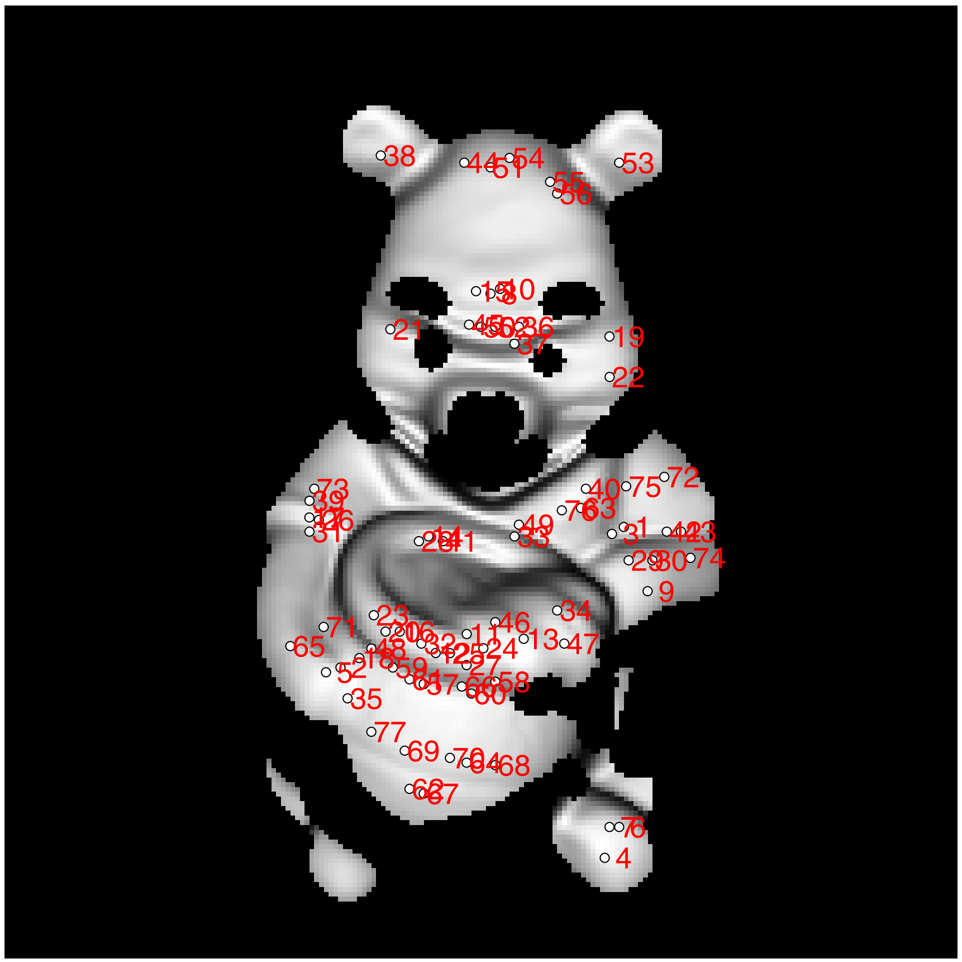


- this is not mutually best matching:
- a vertex with successors (like q) can be in M
  (MBM is a subset of M)
- $O(n^4)$ time complexity for n-point matching
  ($O(n^3)$ algorithm exists)
- Easy to implement
- Can be massively parallelized
  (stability of a network of comparators)
- This algorithm is valid for special class of orientations only, see the paper.

Coarse Range Image Registration: IP Detection

Detection of interest points I_i in each range image i

- all points of good localizability (=dissimilarity to immediate neighborhood)



I_1

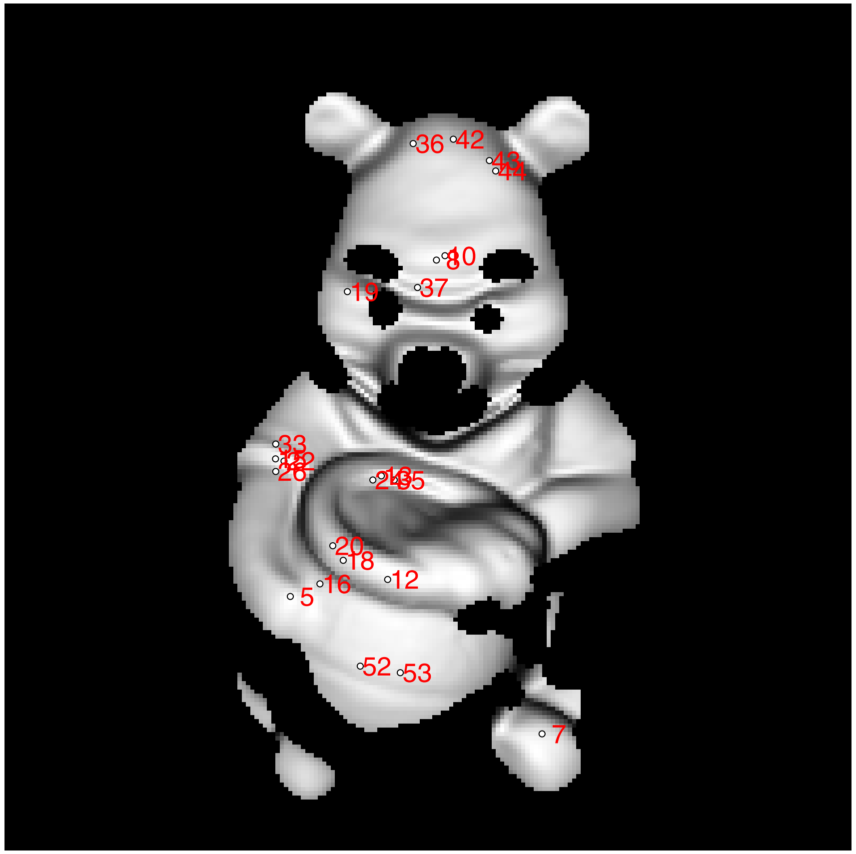
I_2

Coarse Range Image Registration: IP Selection

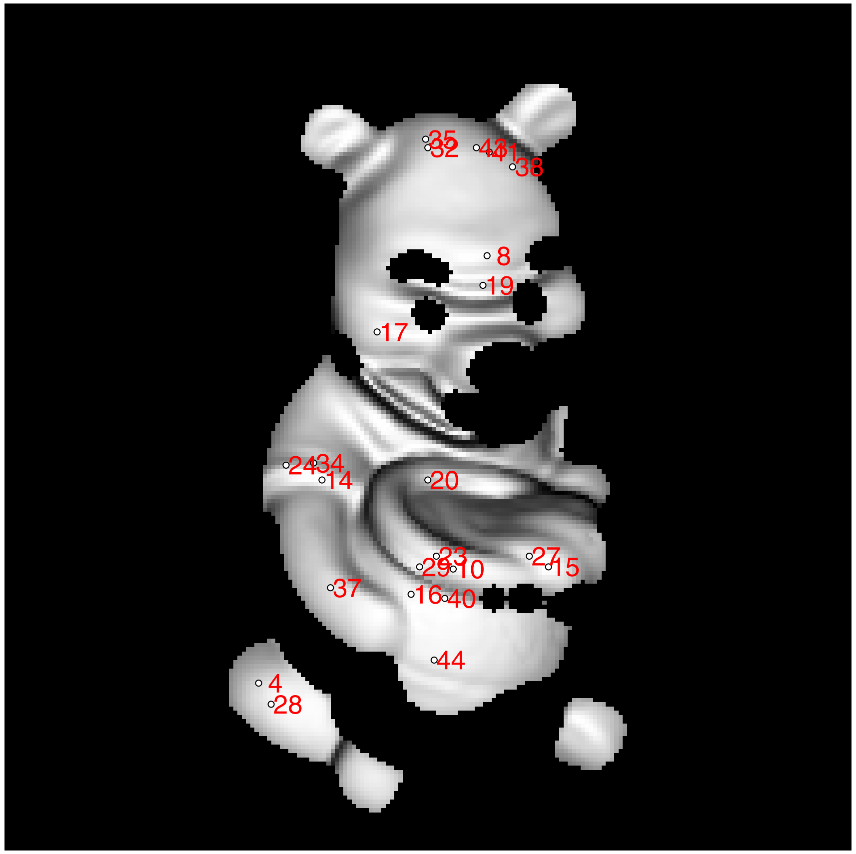
Interest point **selection** in each range image gives I_i^*

- finding mutually geometrically inconsistent subset of I_i

  for each pair $\mathbf{x}, \mathbf{y} \in I_i^*$ there is no allowed rigid transform bringing $\mathrm{ngh}(\mathbf{x})$ onto $\mathrm{ngh}(\mathbf{y})$
- problem size reduction
- improves data rejection rate due to repeated similar structures
- can be found by solving an SSK problem see the paper



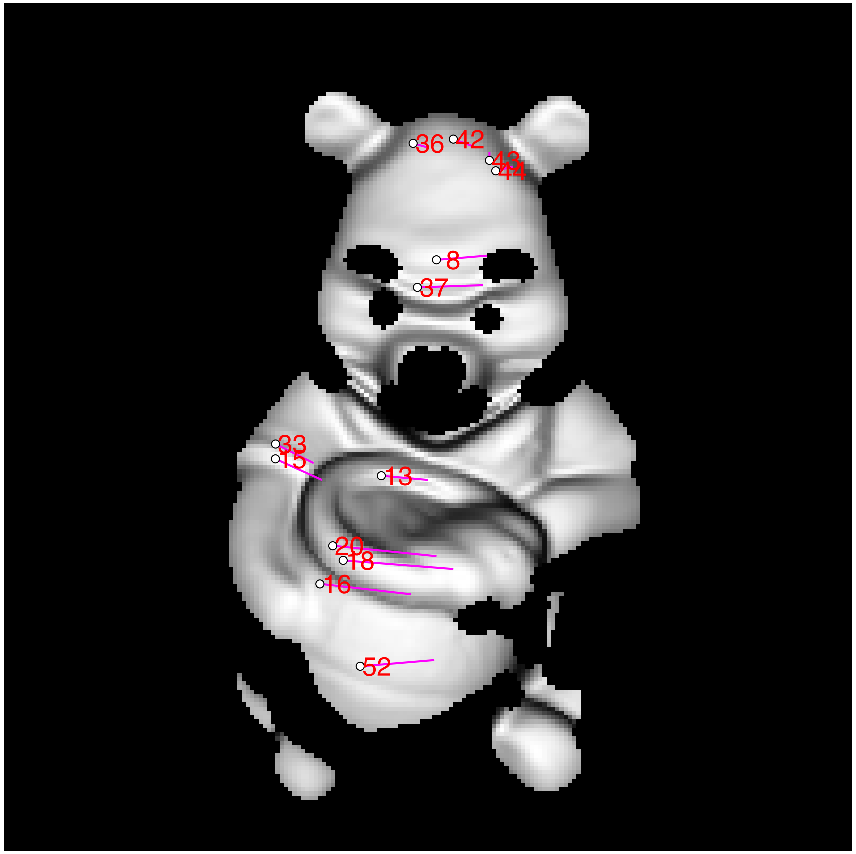
I_1^*

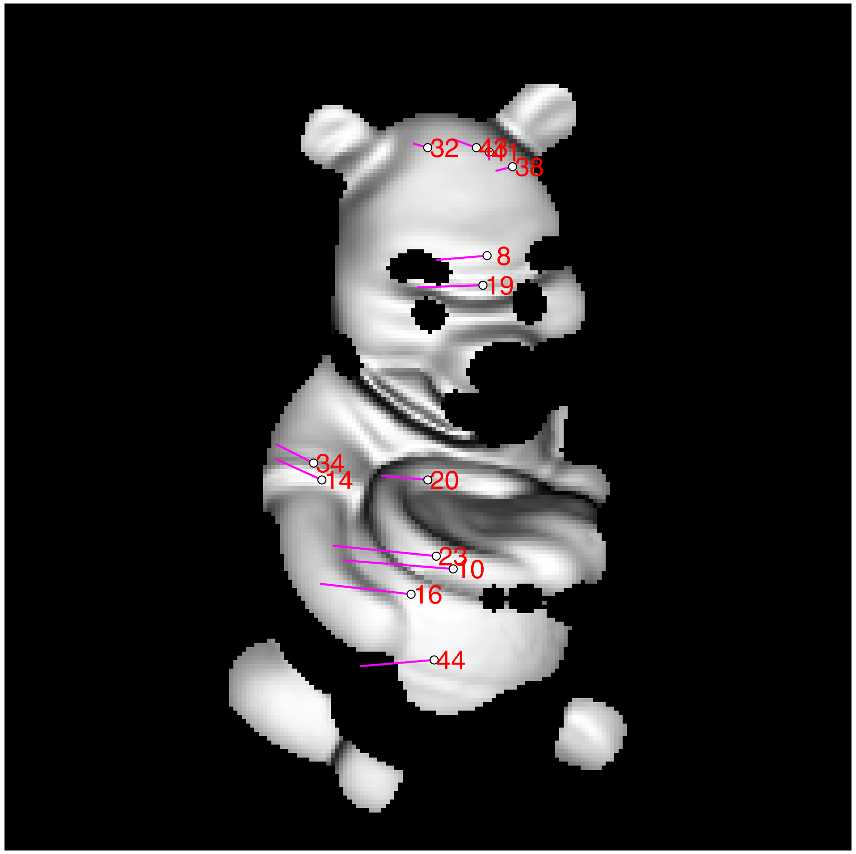


I_2^*

Coarse Range Image Registration: Matching

Matching via SSK as described before.



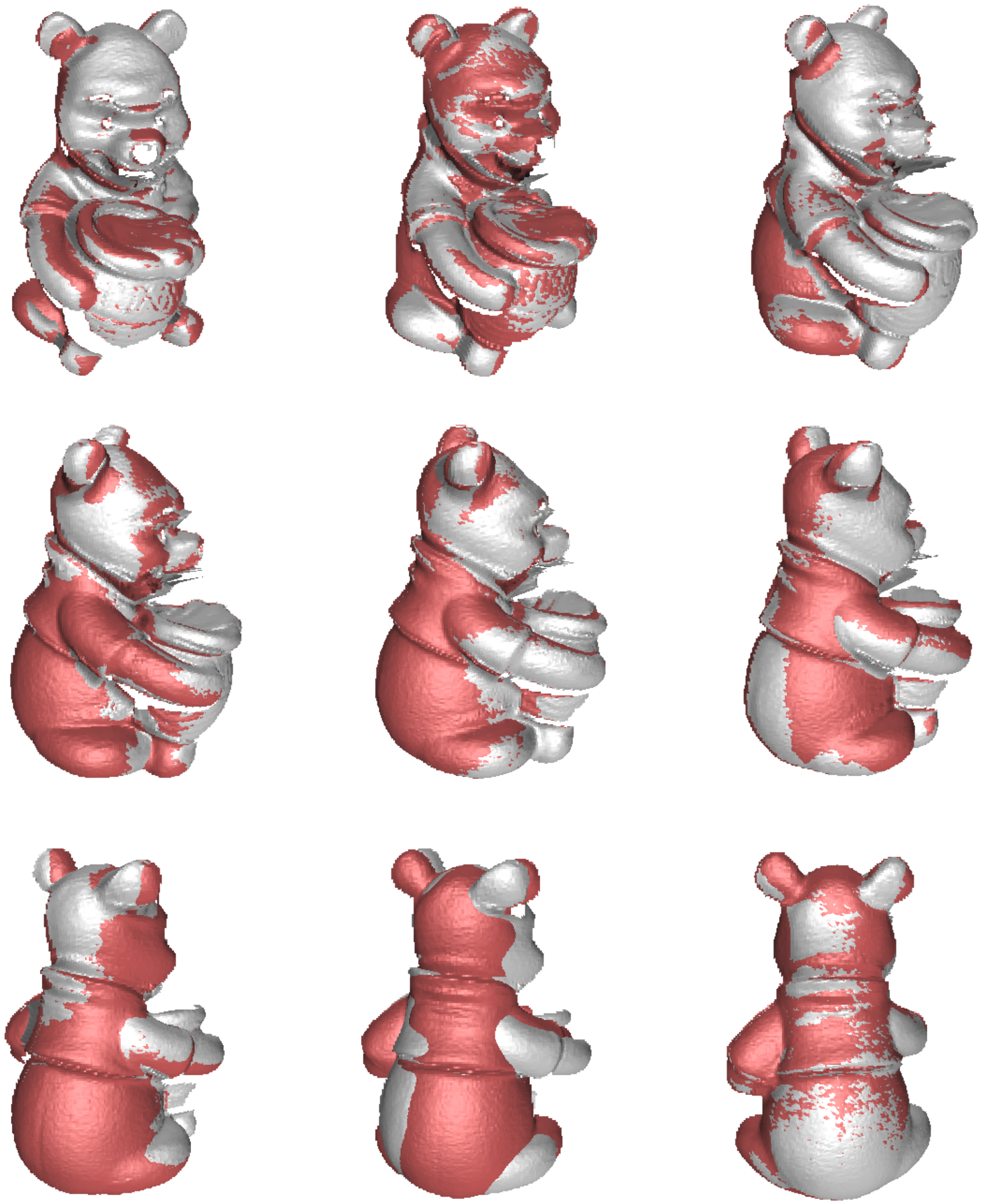


Interpretation of the Matching Procedure

1. Find a consistent match p that clearly correct (as measured by F)
2. Constrain acceptable rigid motions to those consistent with p
3. Repeat

Result: Ever tighter geometric guidance as the similarity decreases.

Results on Pooh Dataset



1 failure

main failure mode: empty matching

Timings (per pair)

normals	0.2 min
features	0.6 min
IP detection	0.1 min
IP selection	1.7 min
matching	0.1 min
total	2.7 min

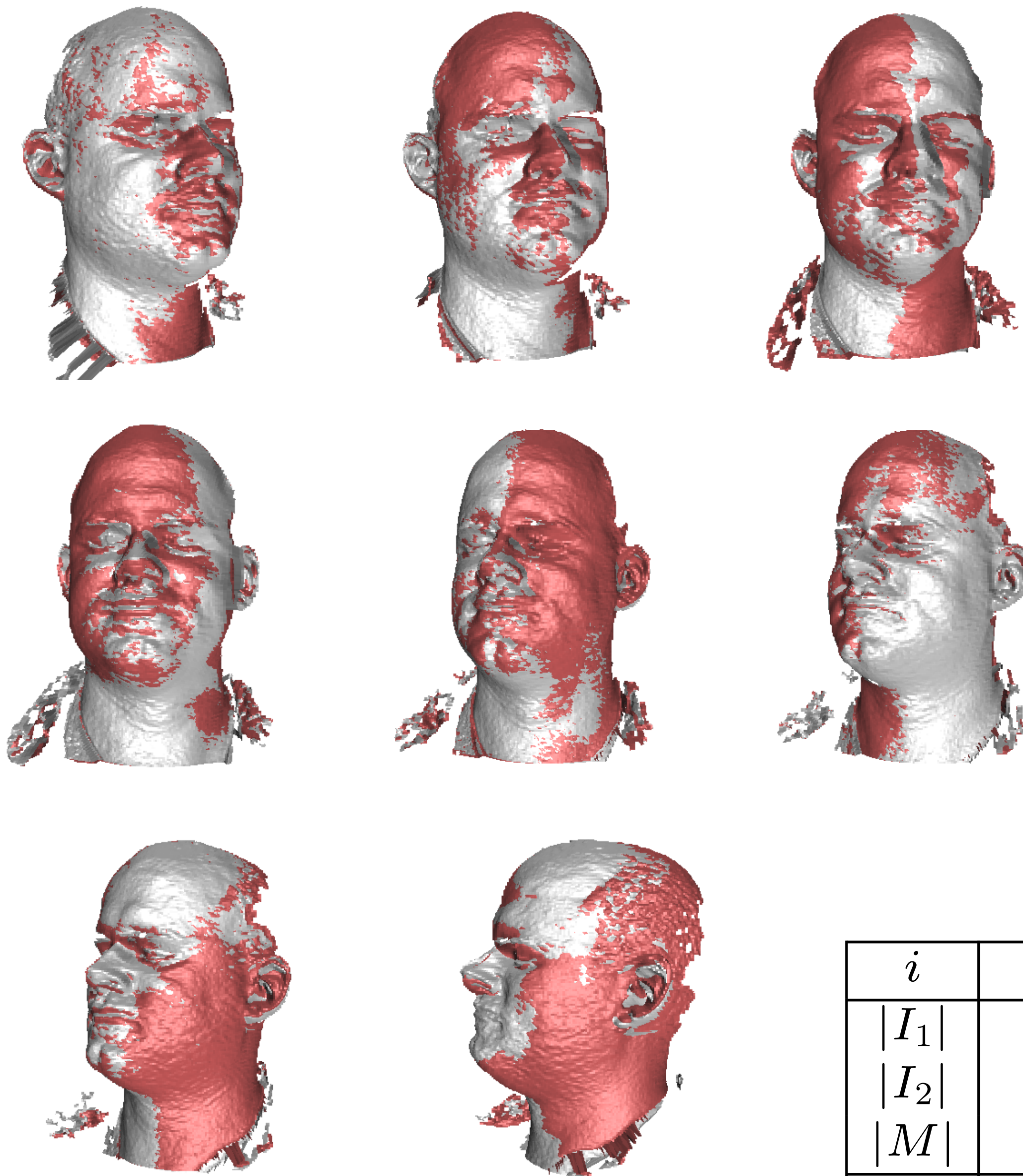
Data courtesy of Ohio State University

Results on Pooh Dataset (cont'd)

coarse registration after ICP refinement

- Coarse registration: rotation well estimated, translation not so well
  (but ICP can deal with it)
- around the neck: occlusion boundary/interreflection artefacts in data

Results on Rick1 Dataset



i	1	2	3	4	5	6	7	8	improvement
$\lvert I_1 \rvert$	37	35	28	34	32	35	31	52	
$\lvert I_2 \rvert$	41	37	28	34	33	32	33	58	
$\lvert M \rvert$	18	17	10	9	14	18	10	11	
ε_0	1.79	5.71	4.72	3.25	9.38	2.51	6.07	8.06	
$\varepsilon_{\mathrm{CR}}$	0.35	0.43	0.96	0.58	0.43	0.45	0.60	1.14	8.4×
$\varepsilon_{\mathrm{ICP}}$	0.24	0.26	0.46	0.39	0.26	0.31	0.37	0.72	1.6×

Conclusions

- SSK can be used for **coarse registration**

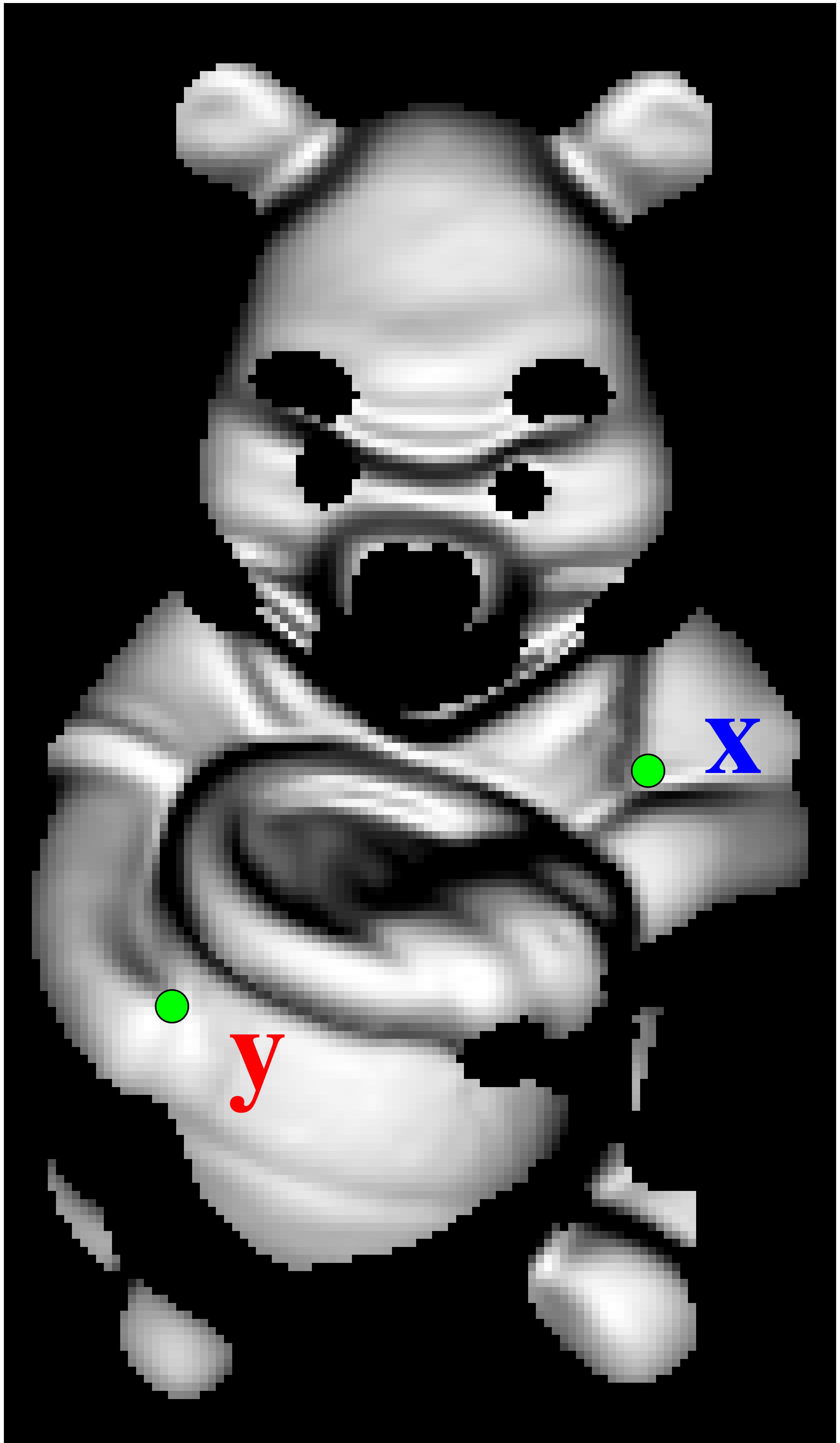
  reduces initial closest-point error by about the order of magnitude

- What does SSK open for us?

  1. **Robust** behavior: either finds a unique robust solution or rejects data.

     robust w.r.t. small data perturbation

  2. **Multi-criterial** matching (e.g. geometry, color) w/o mixing apples and pears.
  3. Algorithmic **simplicity**.


x
y